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LASER TRIANGULATION UNDER STRUCTURED LIGHTING: A DIDACTIC TRAINING SESSION

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Abstract: A training session dealing with triangulation under laser plane lighting is presented. Students are asked to implement an algorithm allowing the measurement of the width of rectangular plane objects. Experimental results obtained by the students show an accuracy of 1%, for objects of different widths (10-40 cm), attitudes and distances from the instruments going up to 2.5 m.

Keywords: optical instrumentation, structured lighting, didactic training session.

1. INTRODUCTION

In the scope of a course given at our university, dealing with sensors and instrumentation, a two-part module related to triangulation under laser plane lighting is proposed to the students. These students are mainly following the fourth university year of the electromechanical engineering diploma.

The first part consists in a two-hour seminar introducing the main concepts of the method, dedicated to optical noncontact dimensional measurements while the second, detailed here, requires from the students, organized in groups of three, to (a) implement an algorithm allowing one particular type of dimensional measurement, (b) calibrate the instrument and (c) present some "basic" results (e.g. in terms of accuracy).

The aim being only to make them familiar with the method, the demanded function is chosen as simple as possible: it consists in measuring the width of a rectangular plane object, supposed infinitely thin and—to not make the exercise too easy—of random orientation against the instrument.

The students are eventually invited to produce a short report about their work. Note that the conception of the instrument and the programming of the image processing algorithm (see below) are not part of the training session.

The work load is globally of about fifteen hours, divided in five stages:

- introduction seminar: 2 hours;
- familiarization with the hardware and the software: 2 hours;
- development and implementation of the chosen method: 4 hours;
- tests and experimentation: 3 hours;
- redaction of the report: 4 hours;

2. METHOD

2.1. Triangulation under Laser Plane Lighting

The method is based on the concept of triangulation under laser plane lighting. It is an optical technique exploiting a strictly geometrical approach [1].

Let the target under test have an arbitrary position with regard to the (x, y) plane of a monochrome charge-coupled device (CCD) camera observing the scene (Fig. 1). The reference orthogonal system (x, y and z) is centered on the optical center of the camera (objective lens), with axis y (defined as the optical axis) perpendicular to the CCD sensor plane and axes x and z parallel to axes x_{pix} and y_{pix} respectively. The camera is PC-connected through an appropriate interface.

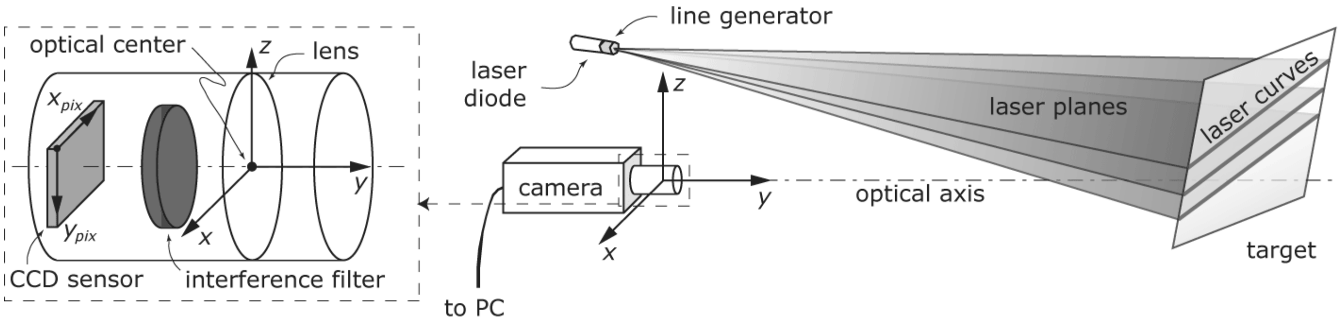

Fig. 1. Configuration of the device. Three-dimensional view.

The object, supposed perfectly planar, rectangular, infinitely thin and of diffuse reflection (at least partially), is illuminated by three laser planes slightly tilted in the *y* direction on the (*x*, *y*) plane of the camera. A set of three straight lines is thus created on the object. We will see that two laser planes are sufficient. Nevertheless, the higher the amount of planes, the better the accuracy by mean effect, but also the computing complexity. A number of three planes seems to be a good compromise. They are generated by a specific line generator placed at the exit of a laser diode, and delivering—at a given distance from the source —a uniform flux density along the plane.

A narrow-band interference filter, centered on the wavelength of the laser diode, is placed between the CCD sensor and the objective of the camera, in order to increase the contrast between the laser lines to be extracted and the rest of the scene. The appropriate analysis of the three laser lines on the image will allow the determination of the object width.

2.2. Parameters of the Instrument

Our instrument has nine geometric parameters, the two first ones concerning the camera and the others the relative position of the laser planes regarding the camera (cf. Fig. 2):

- the two focal lengths f_x and f_y of the camera lens, expressed in number of pixels in the x_{pix} and y_{pix} directions respectively ($f_x = f_y$ if the pixels are square). When the camera is focused on a target at large distance with regard to the focal length (> 10 cm), which we will suppose henceforth, they represent the perpendicular distance between the CCD sensor and the optical center;
- the three distances e_i along axis *z* from the optical center to the laser planes (*i* denotes the number of the laser plane; $i = 1...3$);
- the three angles δ_i with axis *y* of the laser planes viewed in a cut in the (*y*, *z*) plane;
- angle ε (not represented on Fig. 2) with axis *x*, of the laser planes viewed in a cut in the (*x*, *n*) plane, where *n* is the normal vector of the concerned laser plane. Ideally equal to zero, ε is small and supposed identical for the three laser planes (which is appropriate seen the high quality of the laser optics).

Distances e_i are intentionally limited to about 10 cm to maintain the compactness and the ambulatory nature of the instrument. Tilt angles δ_i and the zoom factor of the camera (adjusted mainly by changing the focal length) are chosen so that the whole laser lines are maintained in the field of view of the camera for the distance and width ranges of the objects concerned by the application.

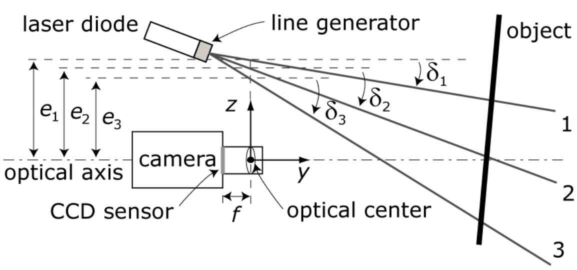

Fig. 2. Geometry of the device. Cut in the (*y*, *z*) plane.

2.3. Image Processing

An image processing algorithm of our own [1], implemented in *C*++ and given to the students, extracts the laser lines intersected by the planar object by isolating them from the rest of the scene. For sake of clarity, the different operations cited in [1] are shortly reminded to the reader.

The interference filter carries out a substantial part of the task, but some additional treatments on the 256-gray-level image are still needed to eliminate the noise surrounding the laser lines. These are: subtraction of two sequentially-taken pictures (one with the laser diode on, the other with the diode off), thresholding, segmentation, laser line selection and column-by-column lines' ordinate calculation (for which we reach a subpixel accuracy by exploiting the generally reasonably horizontal orientation of the laser lines).

The output of this algorithm consists of three lists containing the (x_{pix}, y_{pix}) coordinates of all the pixels belonging to each laser line.

2.4. Link between the Image Plane and the 3-D Scene

A one-to-one correspondence exists between any pixel of the three lines in the image plane (x_{pix},y_{pix}) (i.e. the CCD sensor plane) and the (*x*, *y* and *z*) coordinates of the corresponding point in the three-dimensional (3-D) scene. We suppose that the image we are processing is distortion-free. In Section 2.6 we explain that even if the original image presents some distortion, it can be estimated and corrected before further processing through an appropriate algorithm [2-3].

To determine the bijection, we rely on the camera *pinhole* model, based on central projection. On Fig. 3, concerning the determination of the x_{pix} coordinate of any point *P* in the 3-D space, we obtain the left expression in (1). We apply the same principle on the y_{pix} coordinate, giving us the camera model:

$$x_{pix} = f_x \frac{x}{y};\ y_{pix} = f_y \frac{z}{y} \tag{1}$$

where x_{pix}, y_{pix}, f_x and f_y are pixel numbers. If we now consider the equation of the laser plane *i*

$$x \tan \epsilon + y \sin \delta_i + (z - e_i) \cos \delta_i = 0 \tag{2}$$

we obtain a system of three equations, the solution of which gives us the 3-D coordinates of any point *P* belonging to laser line *i*, from its image coordinates:

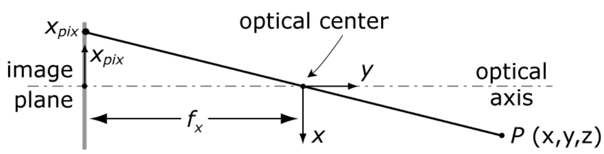

Fig. 3. x_{pix} determination. View in the (*x*, *y*) plane.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \dfrac{x_{pix}\, y}{f_x} \\ \dfrac{f_x f_y e_i \cos\delta_i}{f_x f_y \sin\delta_i + f_x y_{pix} \cos\delta_i + f_y x_{pix} \tan\epsilon} \\ \dfrac{y_{pix}\, y}{f_y} \end{pmatrix} \qquad (3)$$

where y is expressed only once to make short. Note that later we will use respectively small and capital letters for running and particular coordinates.

2.5. Proposed Method for the Width Calculation

With the coordinates of all the pixels belonging to the three laser curves at their disposal, the students can easily compute the 3-D coordinates of the corresponding points in the (x, y and z) coordinate system by (3). They are then requested to solve the problem of measuring the width of the target, by any appropriate method. The programming language has to be C++.

The basic idea is of course to identify the edges of the laser lines, which belong to two supposedly parallel straight lines. The mutual distance of these lines is the searched quantity. The students are then confronted with the unavoidable problem of the imperfect parallelism and straightness of the two reconstructed edges.

We present here a possible approach to tackle this problem. The redundancy is systematically exploited to increase the accuracy by mean effect. The calculation is divided in three steps: (a) calculation of the directing vector of the object's edges delimiting the object's width, (b) determination of the vector expression of each laser segment and (c) projection of those segments on a vector perpendicular to the directing vector of the edges and belonging to the plane of the target. They are described hereafter.

On Fig. 4, the extreme points of each laser curve i are identified as $P_{1,i}$ and $P_{2,i}$. They belong to the two parallel edges delimiting the object's width.

For each edge j (j = 1...2), it is possible to calculate three different expressions of its directing vector $\vec{V}_j$ by

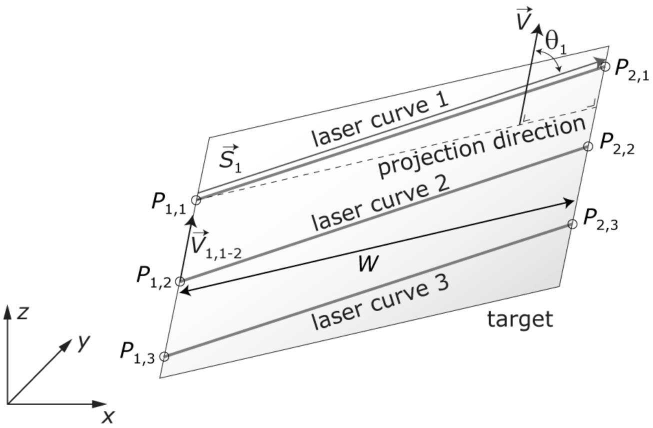

Fig. 4. Geometry of the situation. 3-D view.

considering the different laser plane couples 1-2, 2-3 and 1-3. For instance, for couple 1-2, one has (see also Fig. 4):

$$\vec{V}_{j,1-2} = \frac{\left(X_{j,2}-X_{j,1},\, Y_{j,2}-Y_{j,1},\, Z_{j,2}-Z_{j,1}\right)}{\sqrt{\left(X_{j,2}-X_{j,1}\right)^2+\left(Y_{j,2}-Y_{j,1}\right)^2+\left(Z_{j,2}-Z_{j,1}\right)^2}} \qquad (4)$$

where $X_{j,i}$, $Y_{j,i}$ and $Z_{j,i}$ are the coordinates of point $P_{j,i}$ in the (x, y and z) coordinate system. Notice that $\vec{V}_{j,1-2}$ is normalized.

As the three points of each edge are not strictly aligned, the best expression of the normalized $\vec{V}_j$ is:

$$\vec{V}_j = \frac{\vec{V}_{j,1-2}+\vec{V}_{j,2-3}+\vec{V}_{j,1-3}}{3} \qquad (5)$$

Edges 1 and 2 being ideally parallel, the average of vectors $\vec{V}_1$ and $\vec{V}_2$ represents at best the directing vector $\vec{V}$ (shown on Fig. 4):

$$\vec{V} = \frac{\vec{V}_1+\vec{V}_2}{2} \qquad (6)$$

The accuracy on the—still normalized—directing vector $\vec{V}$ is thus eventually improved by mean effect on six different expressions of it, according to (4).

Each laser segment can now be represented by a vector $\vec{S}_i$ of expression (cf. Fig. 4):

$$\vec{S}_i = \left(X_{2,i}-X_{1,i},\, Y_{2,i}-Y_{1,i},\, Z_{2,i}-Z_{1,i}\right) \qquad (7)$$

These segments are making an angle θ_i, represented on Fig. 4, with the directing vector $\vec{V}$ of the object edges, determined by:

$$\cos\theta_i = \frac{\vec{V}\cdot\vec{S}_i}{\left\|\vec{S}_i\right\|} \qquad (8)$$

Note that the norm of $\vec{V}$ does not appear in the previous expression as it is unitary.

By projecting vectors $\vec{S}_i$ on a vector perpendicular to the directing vector $\vec{V}$ and belonging to the target plane (the projection direction appears on Fig. 4), we obtain three different expressions of the object width W:

$$W = \left\|\vec{S}_i\right\| \sin\theta_i \qquad (9)$$

The accuracy on W is finally improved by mean effect on the three different expressions of it (one for each laser plane i).

2.6. Calibration of the Instrument

The camera and laser plane parameters are determined through two distinct calibration procedures. They are essential because of the critical value of the instrument's parameters.

The first operation, performed on a plane checker pattern, concerns the camera parameters. A few image of the pattern are captured under different aims, and a dedicated algorithm described in [2-3] calculates the camera parameters, i.e. the focal lengths f_x and f_y, the location of the

(x_{pix}, y_{pix}) coordinate system's origin (i.e. the intersection of the optical axis with the CCD sensor) and two distortion coefficient. The last-mentioned permit to correct the distortion of the images used for the measurements.

The second calibration procedure concerns the laser plane parameters. Several measurements are made on a few known plane rectangular objects, at various and a-priori-unknown distances. All necessary data for the width calculation are collected, and an optimization algorithm using gradient descent [4] calculates the laser plane parameters minimizing the mean relative error ρ_W on the objects' widths.

The students are asked to calibrate the camera with a Matlab® toolbox at their disposal [5], and the laser planes / camera system by implementing the adequate *C++* algorithm of their own using gradient descent.

3. RESULTS

We provided the students with a PC-connected pre-prototype composed of a 10-mW 670-nm laser diode crossing a 30°-aperture 3-line generator (interbeam angle 1.5°) and a monochrome CCD camera, the sensor resolution of which is of 576 x 768 pixels, with a theoretically square pixel size of 6.25 µm and an objective's focal length of 8 mm, or 1280 pixels. Vertical distances e_i were fixed to about 10 cm and tilt angles θ_i to 7°, 8.5° and 10°.

As a matter of example, Fig. 5 shows the photograph of a typical scene acquired by the camera. The target is placed in a clear room, in front of a wall. The raw image, obtained before the processing, is on the left. The additional lines of lower luminance clearly visible on the object are due to a grating effect of the laser optics. These lines, together with —where applicable—the segments out of the target, disappear after data processing (right image).

Before proposing the training session to the students, we made a measurement campaign of 50 experiments, in a clear room, on five rectangular objects of different widths (10-40 cm) and at various aims and distances between the operator and the object (up to 2.5 m).

The results we obtained are presented on Fig. 6 showing the histogram of the relative error ρ_W on the width. Table I gives recapitulative figures (the two first lines present the percentage of measurements for which $|\rho_W|$ is below 1 and 1.5% respectively; ρ denotes a relative error and σ a standard deviation). Fig. 7 details the results of 10 measurements at various distances on a 26-cm wide object.

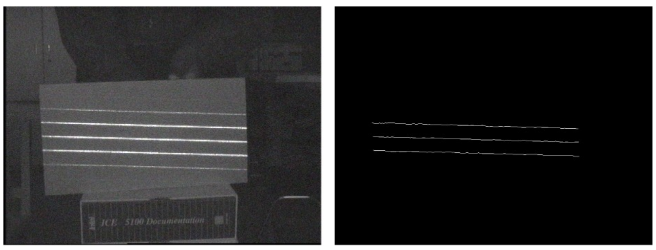

Fig. 5. Example of observed scene. The raw image is on the left, the same picture after image processing being on the right.

These results allow us to claim a general accuracy of 1%.

As for the results obtained by the student, even if they opt for a different method for the width determination, they are generally of similar quality.

4. STUDENT SKILL ACQUISITION

The skills acquired by the students at the end of the training session include: apprehension of a concrete noncontact dimensional measurement problem; accuracy and uncertainties concepts; image processing basics; optimization problem (gradient descent); spatial view; C++ programming; mechatronics integration. The multidisciplinary character of the training session has to be emphasized.

5. CONCLUSIONS

A didactic, multidisciplinary training session has been presented. Intended for students working in the field of sensors and instrumentation, it provides them with an attractive way of familiarizing themselves with the concept of laser triangulation under structured lighting.

Table I. Recapitulative Figures for Rectangular Plane Objects.

$\|\rho_W\| < 1\%$	94%
$\|\rho_W\| < 1.5\%$	100%
$\rho_{W,\text{ mean}}$	-0.05%
$\sigma_{\rho W}$	0.56%

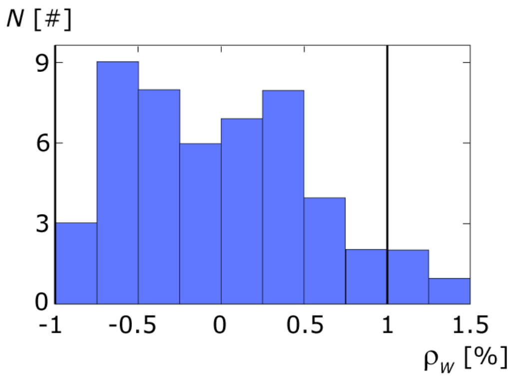

Fig. 6. Histogram of the relative error on the object width.

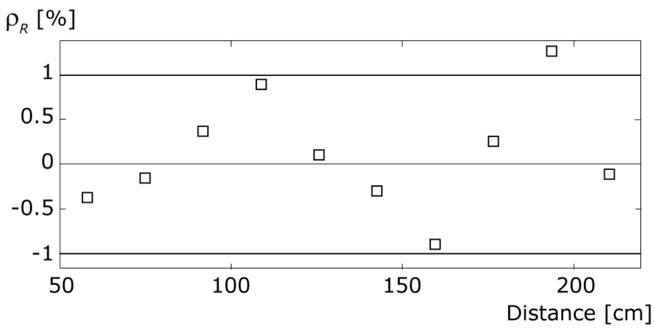

Fig. 7. Results of measurements taken on a 26-cm wide rectangular object at various distances.

The proposed task is to determine the width of a perfectly rectangular, planar and infinitively thin object of random orientation with regard to the instrument. The students are invited to perform the camera calibration, to implement and experiment the laser plane calibration, and to imagine, program and test the width determination method. They are given the instrument and the image processing algorithm. The used language is *C++*.

Generally, the results that they obtain are satisfactory as the relative error on the width is most of the time below 1%, for widths ranging from 10 to 40 cm and for a distance—a priori unknown—between the instrument and the planar object going up to 2.5 m. The acquired skills are numerous.

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