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UNCERTAINTY BUDGET OF PRESSURE BALANCE EFFECTIVE AREA DETERMINED BY COMPARISON METHOD

Davor Zvizdić[1], Lovorka Grgec Bermanec[1], Georges Bonnier[2], Eliane Renaot[2], Tomislav Veliki[1]

[1] LPM-FSB, University of Zagreb, Zagreb, Croatia
[2] BNM-INM / CNAM, Paris, France

Abstract – This paper presents a simplified model for the evaluation of the measurement uncertainty of pressure balance effective area calibrated by method of comparison which is being applied at Laboratory for Process Measurements (LPM). The methodology is applied for oil-operated industrial pressure balances in the gauge pressure range up to 60 MPa. The scope of the procedure is illustrated by calculation example.

Keywords: measurement uncertainties, pressure balances calibration

1. INTRODUCTION

The characterization of the pressure balance requires careful evaluation of all systematic and random factors affecting the pressure measurement. Among the different systematic factors affecting pressure measurements the largest (type B) uncertainty contribution is the determination of the effective area, A_0 at atmospheric pressure and reference temperature. Effective area can be determined using either the "fundamental" or comparison method. A "fundamental" calibration involves having the effective area of the balance determined using only measurements of the SI base units plus a suitable mathematical model. A comparison calibration determines the effective area against another pressure balance of which the effective area is already known. Calibration method used in this paper is comparative one, and it involves the transfer of effective area of one piston/cylinder (P/C) assembly to another utilizing pressure based "crossfloat" technique over a range of pressures.

2. CALIBRATION AND COMPUTING METHOD

A pressure balance under test is hydrostatically balanced against a similar standard of known effective cross-sectional area. The basis of the comparison is determination of the loads at which each balance would individually barostat the system at precisely the same pressure. State of equilibrium is identified by rate-of-fall technique [2], which means that load of each assembly is carefully adjusted by means of trimming weights until both pistons are falling in their natural rates.

"Fig. 1" shows a common system configuration for a single medium hydraulic "crossfloat".

The pressure balance under test, T, is connected to a standard pressure balance, E, and a pressure generator, S. The loads, M, on both pistons are adjusted until they are hydrostatically balanced at the required pressure. Difference in altitude, Δh, between the reference levels of the E and T balances as well as the temperatures of the environment, tested and standard piston-cylinder unit are measured.

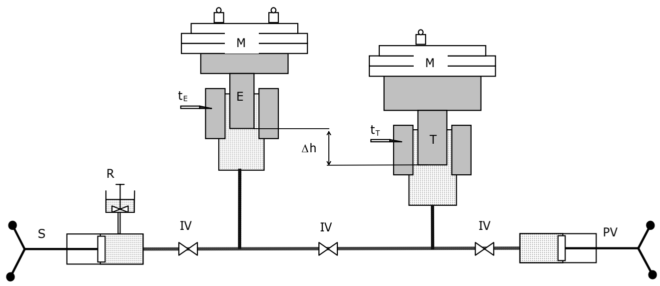

Fig 1. System configuration

Mathematical model used to calculate pressure balance effective area at zero pressure is known as the P method which requires all loading forces acting on the system to be evaluated and summed, including those due to internal 'fluid-head' and buoyancy effects.

For each calibration point the effective pressure, p_e, measured at the reference level of the E is calculated, using the known characteristic of the standard piston/cylinder assembly according to equation (1).

$$p_e = \frac{\left[\sum_i \left[m_i \cdot \left(1 - \frac{\rho_a}{\rho_{mi}}\right)\right] + \left(hA_0 - v\right)\cdot\left(\rho_f - \rho_a\right)\right]_E \cdot g + \Gamma \cdot c_E}{A_{0_E} \cdot \left(1 + \lambda \cdot p_e\right) \cdot \left[1 + \left(\alpha_K + \alpha_c\right)_E \left(t_E - 20\right)\right]} \qquad (1)$$

Where:

- A_{0e} is the effective area of standard P/C unit
- p_e is the gauge pressure measured at the bottom of the piston
- m_i is the individual mass value of i-th weight applied on the piston, including all floating elements
- g is the local gravity
- ρ_a is the density of surrounding air

ρ_{mi} is the density of i-th weight
α_p is the linear thermal expansion coefficient of the piston
α_c is the linear thermal expansion coefficient of the cylinder
t is the measured temperature of the piston-cylinder assembly during its use
λ is the pressure distortion coefficient
ρ_{fl} is the density of the measuring fluid
Δh is the difference in altitude between the reference level of the balance and the point where the pressure has to be measured.

Index $_E$ pertains to standard P/C assembly.

Calculation of air density in “(1)”, ρ_a, is based on measured environment conditions using approximation formula:

$$\rho_a = \frac{0,34848 \cdot p_o - 0,009024 \cdot RH \cdot e^{0,0612 \cdot t}}{273,15 + t_o} \tag{2}$$

where p_o is ambient pressure, t_o is ambient temperature and RH presents relative humidity in the calibration room.

Determination of the effective area, A_{eT}, of the P/C assembly of a tested pressure balance is derived from the equation used to calculate pressure, so for each calibration point A_{eT} can be calculated as:

$$A_{e_T} = \frac{\sum_i \left[m_i \cdot \left(1 - \frac{\rho_a}{\rho_{m_i}} \right) \right]_T \cdot g + \sigma \cdot c_T}{(p_e + \rho_{fl} \cdot g \cdot \Delta h) \cdot \left[1 + \left(\alpha_P + \alpha_c \right)_T (t_T - 20) \right]} \tag{3}$$

Where index T pertains to balance under test.

The effective area is a linear function of the pressure “(4)”. The experimental results from crossfloat calibration have p_e, as the independent variable, x. The dependent variable is $A_{eT}(p_e)$. The aim of calibration is determination of effective area at null pressure A_0.

$$A_e = A_0 \cdot (1 + \lambda \cdot p_e) \tag{4}$$

A graph of the effective areas, A_{eT}, is plotted and the method of Gauss is used to fit a straight line to the results. The gradient gives the difference in the distortion coefficients and the intercept with the abscissa gives A_0, the effective area extrapolated to zero applied pressure.

For n calibration points intercept of the best-fit straight line with the abscissa i.e. A_0 is calculated as:

$$A_0 = \frac{1}{n} \cdot \left[\sum_{i=1}^{n} A_{ei} - b \cdot \sum_{i=1}^{n} p_{ei} \right] \tag{5}$$

where b is the gradient of best fit straight line given by:

$$b = \frac{n \cdot \sum_{i=1}^{n} \left(p_{ei} \cdot Ae_i \right) - \sum_{i=1}^{n} p_{ei} \cdot \sum_{i=1}^{n} Ae_i}{n \cdot \sum_{i=1}^{n} \left(p_{ei}^2 \right) - \left(\sum_{i=1}^{n} p_{ei} \right)^2} \tag{6}$$

3. MEASUREMENT UNCERTAINTY ESTIMATION

Following the ISO [3] and NPL [4.], recommendations sources of uncertainty are divided in Type A and Type B uncertainties.

According to the mathematical model the uncertainties of the effective pressure, $u(p_e)$, are estimated first as Type B, taking account following quantities:

- uncertainty of the masses, $u(M)_E$,
- uncertainty of the standard P/C effective area, $u(Ae)_E$ which comes from the calibration certificate
- uncertainty due to the pressure distortion coefficient, $u(\lambda)_E$, also known from the calibration certificate
- uncertainty due to temperature of the standard $u(\Delta t)_E$, which comes from calibration of thermometer
- uncertainties of thermal expansion coefficients of standard piston and cylinder, $u(2\alpha)_E$,
- uncertainty due to density of the load, $u(\rho_m)_E$
- uncertainty due to air buoyancy, $u(\rho_a)_E$
- uncertainty due to head correction, $u(\Delta h)$
- uncertainty due to surface tension, $u(\sigma)_E$

Most of above mentioned uncertainties are taken from latest calibration report except uncertainty values for thermal expansion coefficient and densities of the loads which are found in literature.

Uncertainties of the masses are calculated as sum of the standard uncertainties of the individual weights because correlation coefficient 1 is assumed:

$$u(M) = \sum_i u_{m_i} \tag{7}$$

After all the uncertainty components have been individually evaluated and converted into standard uncertainties (single standard deviations) they are combined by root-sum-of-squares summation.

$$\begin{aligned} \left(u(p_E) \right)^2 = {} & u^2(\lambda)_E \cdot p^4 + \\ & \left(\begin{aligned} & \left(\frac{u(M)_E}{m_E} \right)^2 + \left(\frac{u(\rho_a)_E}{\rho_{mE}} \right)^2 + \left(\frac{u(A_e)_E}{A_{eE}} \right)^2 + \\ & + \left(2\alpha \cdot u(\Delta t)_E \right)^2 + \left(\Delta t \cdot u(2\alpha)_E \right)^2 \\ & + \left(\frac{\rho_a}{\rho_{m_E}^2} \cdot u(\rho_{mE}) \right)^2 \end{aligned} \right) \cdot p_e^2 \\ & + \left(g \cdot (\rho_{fl} - \rho_a) \cdot u(\Delta h) \right)^2 \end{aligned} \tag{8}$$

After evaluation of the effective pressure uncertainty for each calibration point the uncertainties of the effective area were calculated as Type A and as Type B at the same time. Reason for this decision lies in the fact that only Type A calculation does not include uncertainty of the line pressure and sometimes can result with uncertainties smaller than standard P/C unit have.

Type A components, u_{AeA}, are referred to repeatability of the assembly, estimated as the experimental standard deviation of the estimated mean value of $\overline{A_{eT}}$ and the experimental standard deviation of the gradient.

$$u_{A_eA} = \sqrt{\frac{1}{n-2}\cdot\sum_{i=1}^{n}\left(A_{eT}-\overline{A_{eT}}\right)^2} \qquad (9)$$

Type B uncertainties of effective area, u_{AeB} takes into account uncertainties of the applied pressure u_{pE}, masses u_M, temperature u_t, temperature coefficients $u_{2\alpha}$, and densities of the masses $u_{\rho m}$. Combined standard uncertainty, u_{AeB}, of the effective area of is calculated as:

$$u_{A_eB} = \sqrt{\left(\frac{\partial A}{\partial p_e}\cdot u_{pe}\right)^2+\left(\frac{\partial A}{\partial M}\cdot u_M\right)^2+\left(\frac{\partial A}{\partial t}\cdot u_t\right)^2+\left(\frac{\partial A}{\partial 2\alpha}\cdot u_{2\alpha}\right)^2+\left(\frac{\partial A}{\partial \rho_m}\cdot u_{\rho m}\right)^2} \qquad (10)$$

$$= \sqrt{\left(\left(\frac{u_{pe}}{p_e}\right)^2+\left(\frac{u_M}{M}\right)^2+\left(2\alpha_T\cdot u_{tT}\right)^2+\left(\Delta t\cdot u_{2\alpha}\right)^2+\left(\frac{\rho_a}{\rho_{mT}^{\ 2}}\cdot u_{\rho mT}\right)^2\right)\cdot A_{eT}^{\ 2}}$$

where A pertains to A_{eT}. Finally we have n (n is number of calibration points) u_{A_eB} (Type B) uncertainties and one Type A information, so final uncertainty of effective area at atmospheric pressure and reference temperature, u_{A0}, is assumed to be maximum of these values :.

$$u_{A_0} = \max\left\{u_{A_eB,i}, u_{A_eA}\right\} \qquad (11)$$

The expanded uncertainty U_{A0} is directly derived from the combined standard uncertainty by multiplying it by a coverage factor k=2.

4. RESULTS

Example results are given for oil-operated P/C assembly with 1 to 60 MPa measurement range manufactured by Pressurements. Comparison is performed with LPM standard oil-operated P/C assembly manufactured by Budenberg. Basic data for both instruments and calibration conditions are given in Table I.

Effective area calibration results with their expanded uncertainties are illustrated in "Fig. 2" and Table II. Complete uncertainty estimation, which must be calculated for each calibration point separately, is given in details for one calibration point: p=30 MPa (highlighted row in table II) in Tables III and IV.

Tables II and III creates one body and result in Table II is uncertainty of pressure p_e in pascals which is the largest contributor in Table III (first row) with uncertainty of effective area, u_{AeT} in m^2 as result.

TABLE I Basic data for standard and tested unit

Tested pressure balance	
Type of pressure balance tested	Pressurements S111
Approximation A_0 of test instrument	$4{,}03\cdot10^{-6}$ m^2
Approximation λo f test instrument	$0{,}4\cdot10^{-5}$ MPa^{-1}
Volume for buoyancy correction	(320 ± 2) mm^3
Min operating pressure	1 MPa
Max operating pressure	60 MPa
Piston/Cylinder material	Steel/Steel
Standard pressure balance	
Type of standard pressure balance	Budenberg/ 380D
Effective area of standard instrument	$(0{,}806515 \pm 0{,}000053)\cdot10^{-5}$ m^2
Distortion coefficient of standard instrument	$(3{,}3 \pm 0{,}4)\cdot10^{-6}$ MPa^{-1}
Volume for buoyancy correction	-
Min operating pressure	1 MPa
Max operating pressure	60 MPa
Piston/Cylinder material	Steel/Steel
Calibration conditions	
Reference temperature for A_0	20°C
Difference in height of E and T	$(3{,}3 \pm 0{,}1)$ mm

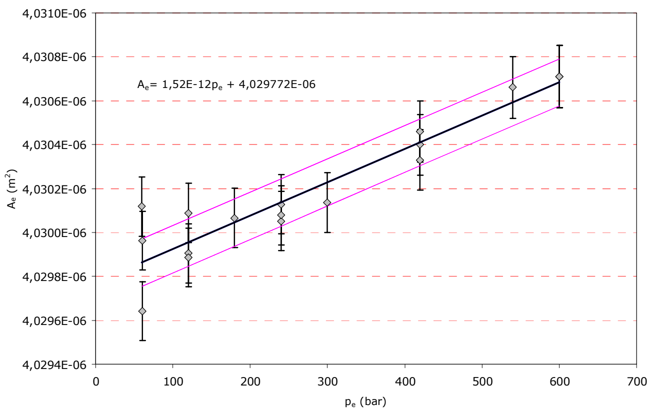

Fig 2. Determination of effective area

TABLE II Calibration results

No	Total Mass on E	Total Mass on T	Temperature on E	Temperature on T	Effective pressure	Effective area A_0	Uncertainty of A_0
	kg	kg	°C	°C	bar	m^2	m^2
1.	4,93502	4,93502	23,31	23,18	59,998482	4,030118E-06	2,69E-10
2.	9,87023	9,87023	23,16	23,23	119,997040	4,029905E-06	2,69E-10
3.	14,80467	14,80467	23,13	23,25	179,983626	4,030066E-06	2,69E-10
4.	19,73974	19,73974	23,09	23,35	239,975687	4,030129E-06	2,70E-10
5.	24,67461	24,67461	23,05	23,37	299,963096	4,030137E-06	2,72E-10
6.	34,54347	34,54347	22,90	23,16	419,920898	4,030400E-06	2,76E-10
7.	44,41209	44,41209	23,02	23,23	539,864002	4,030662E-06	2,81E-10
8.	49,34652	49,34652	22,61	23,01	599,839440	4,030709E-06	2,84E-10
9.	49,34632	49,34632	22,45	23,06	599,839121	4,030711E-06	2,84E-10
10.	34,54347	34,54347	22,61	23,09	419,923577	4,030330E-06	2,76E-10
11.	19,73964	19,73964	22,54	23,08	239,977375	4,030052E-06	2,70E-10
12.	9,87093	9,87093	22,54	23,10	120,007186	4,029885E-06	2,69E-10
13.	4,93632	4,93632	22,54	23,12	60,015301	4,029963E-06	2,69E-10
14.	4,93632	4,93632	22,60	23,12	60,015222	4,029642E-06	2,69E-10
15.	9,87043	9,87043	22,60	23,13	120,000950	4,030089E-06	2,69E-10
16.	19,73984	19,73984	22,53	23,16	239,979859	4,030078E-06	2,70E-10
17.	34,54297	34,54297	22,64	23,16	419,917222	4,030460E-06	2,69E-10
					A_0=	**4,029772E-06**	
					u_{A_0B} =		2,84E-10
					u_{A_0A} =		1,07E-10
					u_{A_0} =		2,84E-10

TABLE III Uncertainty budget for one calibration point (p=30 MPa)

Influence quantity	Numerical value	Uncertainty of the influence quantity	Factor	Standard deviation	Numerical value of the sensitivity coefficient	Contribution to the standard uncertainty of effective pressure
Ae_E	8,06515E-06 m^2	5,30E-10	2	2,65E-10	3,72E+12 MPa/ m^2	986 Pa
t_E	23,05 °C	0,02	2	0,01	659,92 MPa/°C	7 Pa
λ_E	3,30E-07 MPa^{-1}	4E-08	2	2,0E-08	9,0E+09 MPa^2	180 Pa
ρ_a	1,15 kg/m^3	0,03	2	0,015	3797,0 MPa ·m^3/kg	57 Pa
$(\Sigma m)_E$	24,6746115 kg	3,7E-05	2	1,85E-05	1215675 MPa/kg	23 Pa
ρ_m	7900 kg/m^3	100	$\sqrt{3}$	57,74	0,553 MPa ·m^3/kg	32 Pa
Δh	0,03 m	0,003	2	0,0015	8373,13 MPa/m	13 Pa
$(\alpha_p+\alpha_c)_E$	2,20E-05 °C^{-1}	2E-06	$\sqrt{3}$	1,15E-06	9,15E+07 MPa·°C	106 Pa
σ	0,03 N/m	0,003	$\sqrt{3}$	1,73E-03	1247 MPa m/N	2 Pa
p_{eE}=	29,9963 MPa				u_{pe}=	1010 Pa

TABLE III Effective area uncertainty budget for one calibration point (p=30 MPa)

Influence quantity	Value	Uncertainty of the influence quantity	Factor	Standard deviation	Numerical value of the sensitivity coefficient	Contribution to the standard uncertainty of effective area
p_e	29996300 Pa	1010	1	1010	1,3434E-13 MPa/ m^2	1,357E-10 m^2
t_T	13,37 °C	0,02	2	0,01	8,8655E-11 MPa/°C	8,865E-13 m^2
ρ_a	1,15 kg/m^3	0,03	2	0,015	5,1010E-10 MPa^2	7,651E-12 m^2
$(\Sigma m)_T$	12,3287407 kg	1,85E-05	2	9,246E-06	3,2686E-07 MPa $\cdot m^3/kg$	3,022E-12 m^2
ρ_m	7900 kg/m^3	100	√3	57,74	7,4255E-14 MPa/kg	4,287E-12 m^2
$(\alpha_p+\alpha_c)_T$	2,2E-05 $°C^{-1}$	2,0E-06	√3	1,1547E-06	1,2089E-05 MPa $\cdot m^3/kg$	1,396E-11 m^2
A_{eT}=	4,03014E-06 m^2				u_{AeT}=	1,360E-10 m^2
					$U_{AeT=}$	2,72E-10 m^2

Overall uncertainty of effective area at calibration point 30 MPa is 2,7E-10 m^2. Last column in Table II shows uncertainties for all points and it can be seen that they are all alike, and maximum value 2,84E-10 m^2 is accepted as final result.

5. CONCLUSIONS

The problems connected with calibration of pressure balances by method of comparison and estimation of measurement uncertainties were treated theoretically and experimentally in this paper. The aim of proposed procedure is to develop procedure for traceable calibration of pressure balance effective area.

Fundamental characteristics of high–pressure standard measuring equipment, on which all measurements were performed, was described. Theoretical basis of calculation methods for determination of effective pressure, effective area and other characteristics were adjusted to real-life pressure standards.

Starting point in calculation of measurement uncertainty was definition of mathematical model which describes measured quantity i.e. effective area. Major sources of uncertainty were taken into account and grouped into uncertainties of the standard, of the method and of the piston-cylinder unit under test. Effective area calibration results and uncertainty budget for one calibration point of effective area was given. From the results it can be seen that major influence on overall uncertainty of effective area is introduced by standard piston-cylinder unit but depending on the calibration laboratory potentials neither one of influencing quantity mentioned in the budget should not be ignored.

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Authors: Davor Zvizdic, LPM-FSB, University of Zagreb, Ivana Lucica 5, 10000 Zagreb, Croatia, phone: +385 1 6168 333, fax: +385 1 6118 714, davor.zvizdic@fsb.hr;
Lovorka G. Bermanec, LPM-FSB, University of Zagreb, Ivana Lucica 5, 10000 Zagreb, Croatia, phone: +385 1 6168 330, fax: +385 1 6118 714, lovorka.grgec@fsb.hr;
Georges Bonnier, BNM-INM / CNAM, 292, Rue Saint-Martin, 75141 Paris, France, phone: +33 1 402 721 58, fax: +33 1 427 137 36, bonnier@cnam.fr;
Eliane Renaot, BNM-INM / CNAM, 292, Rue Saint-Martin, 75141 Paris, France, phone: +33 1 402 720 21, fax: +33 1 427 137 36, renaot@cnam.fr;
Tomislav Veliki, LPM-FSB, University of Zagreb, Ivana Lucica 5, 10000 Zagreb, Croatia, phone: +385 1 6168 330, fax: +385 1 6118 714, tomislav.veliki@fsb.hr.