XVII IMEKO World Congress
Metrology in the 3rd Millennium
June 22–27, 2003, Dubrovnik, Croatia

NEW WAY OF ACCURACY IMPROVEMENT FOR THE PNEUMATIC DEADWEIGHT TESTER V1600

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Abstract - The V1600 is a compact portable pressure balance suitable for applications where a large number of high accuracy calibrations in ranges from 20 Pa up to 16 kPa, need to be performed on a daily basis.

Calibrations performed at VNIIMS, LNE, MIKE, NIST and PTB have shown the uncertainty of the V1600 is less than 0.1 Pa below 500 Pa and 0.02% of reading from 500 Pa to 16 kPa.

An important part of the uncertainty of the instrument within the range 500 Pa to 16 kPa has been caused by a methodical error of the procedure applied to its calibration until now.

This paper presents the original manner of detection and elimination of the above-mentioned systematic error.

This new calibration approach is based on the equality of Eulerian criterion of the actual device with its mathematical model and allows an increase of the calibration accuracy with approximately 1,5 – 2 times for pressures above 500 Pa.

Key words: deadweight tester, accuracy, calibration.

Introduction

A full description of V1600 is given in [1]. The main instrument component is a transducer, which transforms the gravitational force Mg of the piston and loads into a pneumatic pressure P using a system consisting in a variable throttle (piston-nozzle assembly) and a fixed throttle (the gap between nozzle and nozzle-body).

The output pressure of the transducer is applied to the inlet of a two-cascade regulator, which has the role to support the piston with high accuracy at a particular height above the edge of nozzle.

As the air flows through the gap between the piston cone and the nozzle, a reactive feedback force R is generated.

Therefore the measurement equation of V1600 pressure balance can be written as follows:

$$P = Mg\,(1 - \rho_a/\rho_m)/A_0 + R/A_0, \qquad (1)$$

where: $(1 - \rho_a/\rho_m)$ – air buoyancy correction factor,
$R = G\,(\upsilon Cos\beta - \upsilon_1)/A_0$,
A_0 - geometrical area of the nozzle's edge,
G - mass flow rate,
υ and - υ_1 velocity of the gas exhausting from the nozzle-piston gap and respective velocity of the gas in the space under the piston,
h – effective gap between the piston and nozzle in a perpendicular direction of the velocity υ,
2β - angle at the peak of the piston cone.

The most convenient way to use the V1600 pressure balance is to know its main parameter, the effective area:

$$A = A_0\,(1 - q), \qquad (2)$$

where: $q = G\,(\upsilon Cos\beta - \upsilon_1)/Mg$ which can be re-written:
$q = (4hCos\beta)/(R_0 Eu)$, as $\upsilon_1 << \upsilon$,
R_0 - radius of the nozzle,
$Eu = 2P/\rho\upsilon^2$ - Euler's criterion,
ρ - density of gas.

During the measurement, the piston floats and it is changing its relative position to the nozzle causing a variation of the ratio h/R_0. This is the main reason of nonlinearity of the effective area.

To reduce the variation of the ratio h/R_0, the ideal regulator will change the pressure $P_s = (p_s - p_a)$ before the fixed throttle of the transducer which is acting as its supply pressure accordingly with the below equation. This equation is resulting from the equality of the flow rate through the throttles of the pneumatic transducer when a quadratic dependence of the flow rate with the pressure drop is considered:

$$p_s = (p)^2\,[\alpha^2/(p)^{(k+1)/k} - \alpha^2/(p)^2 + 1/(p)^{(k-1)/k}]^{k/(k-1)} \qquad (3)$$

where: p_s– absolute pressure corresponding to P_s,
p_a = 101325 Pa,
α - the ratio of the effective areas of the throttles (variable to fixed), which is usually the ratio between the proportionality coefficients of the mass flow rate of the gas and the pressure drop on the throttles.

However the coefficient α is a monotone increasing function of the absolute pressure of the gas. Therefore the non-linearity of the effective area A for an ideal V1600 device changes by around 0,6%.

The effective area of V1600 can be determined by direct comparison with a pressure standard of a higher accuracy.

The problem arises when it is necessary to calibrate or to test pressure instruments with an uncertainty less than (0,05 – 0,1) Pa in the range below 3000 Pa, where very often it is not possible to determine the effective area by direct comparison.

Therefore the effective area of each V1600 is determined accordingly with the original method of calibration described in [2]. This method is based on definition of effective area A_c, at a pressure P_c, corresponding to the mass M_c (including piston, carrier and weights), within its range, which is accessible to the available standard, for example: 3, 6 or 16 kPa.

The equation of calibration is given by:

$$A_j = A_c\,[1 - q_c(q_c/q_j - 1)] \tag{4}$$

where: $q_c = G_c\, \upsilon_c Cos\beta/M_c g$,

$\upsilon_c = \{[2kRT/(k-1)]*[1 - (p_a/p)^{(k-1)/k}]\}^{1/2}$ or

$\upsilon_c = (2P/\rho)^{1/2}$ as in the range from 0,02 up to 16 kPa the compressibility of the air can be neglected,

$q_j/q_c = \{(p_j/p_c)^{1/k}\, [(P_{sj} - P_j)/(P_{sc} - P_c)]*(M_c\, M_j)\}^{1/2}$

p_j and p_c – absolute pressure corresponding to the pressure P_c~$M_c g/A_0$ and P_j~ $M_j g/\, A_0$, P_{sj}, as long as the real value of A_0 is unknown,

R - universal gas constant,

T - absolute temperature,

k – adiabatic index.

The analysis of the calibration results from several instruments has shown that the relative accuracy, due to calibration method, begins to increase at 700 Pa and reaches up to 0,009 % at 16 kPa.

The purpose of this paper is to present the original manner of detection and elimination of above mentioned systematic error.

New way of accuracy improvement for the V1600

This behavior of the calibration results can to be explained by both local and travelling losses, which inevitably arise at measuring pressure P_{sj} at its origin place.

The essential losses are created by the connector found between the manometer for measurings Ps and the pipeline.

The inlet hole of the connector should be perpendicularly to an interior wall of the pipeline channel.

If the inlet hole of the connection has a very small diameter (a few tenths of a millimetre), the gas flow will not generate any false pressure readings.

In other case the hole causes some deflections of the stream. Therefore the real static pressure P′ differs from the measured pressure P_s: $P' = P_s \pm \chi\rho\upsilon'^2/2$, where: $\rho\upsilon'^2/2$ – the dynamic pressure due by medial velocity υ' in the pipe channel and χ - is a coefficient, depending on the diameter of the hole and of its shape.

At $\chi > 0$ the gas stream can create small refraction or contrariwise back pressure when $\chi < 0$.

The core of the method used to eliminate the calibration‘s methodical error consists in the followings.

The main component of V1600 is a divider of the supply pressure P_s acting on its inlet. The dependence between theoretical output pressure P of the divider and pressure P_s on its inlet can be given by:

$$P_s = \theta P, \tag{5}$$

where: θ -the relationship of throttles conductivity.

The value of coefficient θ depends on the modes of the gas flow through the throttles and can be determined from equality of the flow rate through the throttles.

The fixed throttle of the divider is manufactured as an annual gap between the nozzle and the nozzle-body with the nominal value of the width H = (0,15 – 0,17) mm and the length L =10 mm, Fig. 1.

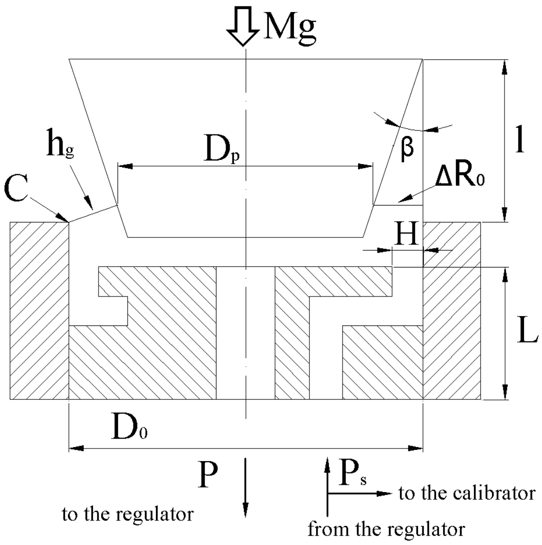

Fig. 1. Schematic drawing of the V1600 pneumatic transducer

The motion becomes turbulent, when the number Re achieves the magnitude determined by expression $Re \sim L/H$ [3]. Hence in the above case, the turbulent mode occurs approximately at $Re \sim$ (60…70).

Therefore the fixed throttle of V1600 is acting as so-called mixed type throttle, in which the mode of motion becomes turbulent at a small supply pressure.

The pressure losses given by the local resistances on the inlet and output of the mixed type throttle have a similar value with the one given by the friction losses.

The total Poiseuille's law allows to take into account both sources of the pressure losses and can be written as follows:

$$(Ps - P)/(\rho\upsilon_2^2)/2 = \lambda\,(L/2H) + \xi_{in} + \xi_{out} + \vartheta + K' \tag{6}$$

where: $\upsilon_2 = Q/(\pi D_0 H)$ - the mean velocity through the channel of the fixed throttle,

$Q = G/\rho$ - the gas flow,

$\lambda = 96/Re$ - coefficient of the pneumatic friction, for $R_0/(R_0 - H) \sim 1$ the coefficient is equal to 96,

ξ_{in} and ξ_{out} - coefficient of irreversible losses at the inlet of the throttle and respectively at the output of the throttle,
ϑ- coefficient of the kinetic energy, as the fixed throttle is between the two chambers (the space before and after throttle) which have a considerable bigger size in comparison with the throttle,
K'= 0,16 - irreversible losses on an initial site of fixed throttle [4].

The left term of the (6) represents a dimensionless value and it is similar with Eu criterion (Euler number). The right term of (6) is practically an extension of a similar criterion for the given physical quantities, as it is equal to the ratio of the two criteria: parametric and Reynolds.

The mode of a motion through a variable throttle is always turbulent due to the piston's oscillations, as it is self-balanced and self-centered continuously.

Proceed from above-mentioned and take into account, that in a steady stated mode the mean velocity in the channel of the fixed throttle is related to the mean velocity of outflow from the nozzle by the equation:

$$\upsilon_2 = \upsilon(\mu h_g/H), \tag{7}$$

where: $\upsilon = (2P/\rho)^{1/2}$ - theoretical velocity of the outflow from the nuzzle,
$\mu h_g = h$ - effective gap between the piston and nozzle in a perpendicular direction to the velocity υ,
$\mu = \varepsilon\varphi$ - coefficient of outflow of the nozzle,
ε - gas compressibility coefficient ,
$\varphi = 1/(\alpha_0 + \zeta)^{1/2}$ –velocity coefficient,
α_0 – correction due by the velocity non-uniformity on the cross section of the channel,
ζ - the nozzle flow resistance,
h_g – geometrical gap between the piston and nuzzle in a perpendicular direction to velocity υ,

the dependence between the supply pressure P_s and the output pressure P of the V1600 divider, as a similarity criterion, can be given by:

$$(Ps - P)/P = (\mu h/H)^2 * [\lambda\,(L/2H) + \xi_{in} + \xi_{out} + \vartheta + K'] \tag{8}$$

or

$$P_s = P[1 + (\mu h/H)^2 * (A + B)] \tag{8a}$$

where: $A = 24(\pi D\nu/Q)*(L/H)$
$B = (\xi_{in} + \xi_{out} + \vartheta + K')$.

At B =0 and when the compressibility of gas can be neglected, the dependency between the pressure drop and the flow rate is given by Poiseuille's formula, which is deduced from the total Poiseuille's law:

$$G = \pi DH^3\Delta P/(12\nu L_1), \tag{9}$$

where: ΔP – the friction losses when it is a laminar flow in the throttle,
$L_1 < L$ – a part of the throttle length in which the velocity in any cross section of a throttle varies under the parabolic law,

and (8) becomes:

$$P_s = P[1 + (12\nu L_1\mu h_g/H^3)*(2/\rho P)^{1/2}] \tag{10}$$

As it is visible from (8) and (10) the pressure difference between the both sides of the fixed throttle *(Ps - P)* depends on a coefficient of outflow.

The coefficient of outflow takes into account all the departures from the theoretical model of the outflow.

As the coefficient of outflow μ is equal to $\varepsilon\varphi$, its functional dependence is very complex. It is influenced by all factors, to which are both dependant, the coefficient of contraction ε and the coefficient of velocity φ (e. q. an initial pressure P_s when on the nozzle is no piston, size of the gap h_g and its form, etc).

Usually the coefficient of outflow is defined experimentally and is calculated with the following equation:

$$\mu = Q/[\pi D_0 h_g(2P/\rho)^{1/2}], \tag{11}$$

where: $h_g = (D_0 – D_p)/(2Cos\beta)$,
D_p - diameter of the piston in the section defined by the perpendicular originated from the point "C" with the slant height of the piston's cone as is shown by the Fig. 1.

Therefore the effective gap between nozzle and nozzle-body of the V1600 can be calculated using the following equation: $h = \mu h_g = Q/[\pi D_0(2P/\rho)^{1/2}]$.

The flow rate was measured by rotameter type KDG11with an expanded uncertainty of 0.03 l/min.

The typical dependence between the designed, the actual ratio α of the effective areas of the throttles and the Reynolds number of the incident flow is shown in Fig. 2.

As it is visible from Fig. 2. at a Reynolds number Re=28,2, the design and the actual coefficient α are equal to α = 0,245 and α = 0.213 respectively and the non-linearity of the diagram achieves the peak value which corresponds to output pressure 700 Pa.

In the range of pressure from 700 up to 3000 Pa the design coefficient α is decreasing monotonically and becomes equal to α = 0,173 at Re = 58,8 which is the theoretical boundary between the laminar and turbulent mode.

At an output pressure of 16 kPa, the Reynolds number is equal to Re = 207,7 and the design and the actual coefficient α becomes α = 0,150 and α = 0,342 respectively.

Thus it is possible to suppose the existence of a quadratic dependence between the differential pressure and flow rate in the range from 3 up to 16 kPa in which case the dependence between pressure supply P_s and output pressure P is given by (3).

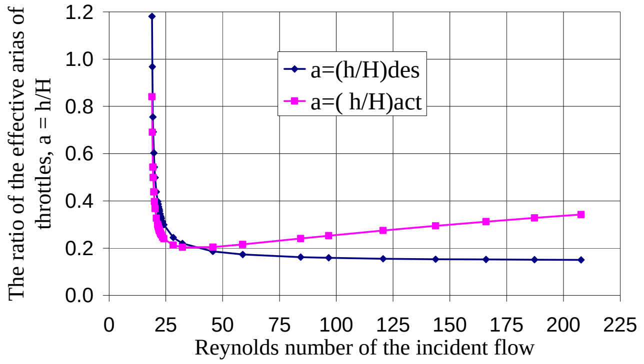

Fig. 2. V1600 s/n 359. Dependence of the ratio α on a Reynolds number of incident flow

If an ideal and an actual device have the same value of the Euler number, they are dynamically similar. The equality of Euler number in dynamically similar systems ensures the similarity of the pressure forces.

The calibration procedure states the necessity of measuring the pressure *Ps* with respect to the relevant values of the output pressure *P*. From these values the coefficient θ of the actual instrument can be determined by linear interpolation: $Ps = (aP + b)$, as is shown graphically in Fig.3. Then, varying the coefficient α for the (3), using the special software, P_s is obtained (5) of an ideal transducer with the same value of the slope: $Psid = (aP + bid)$.

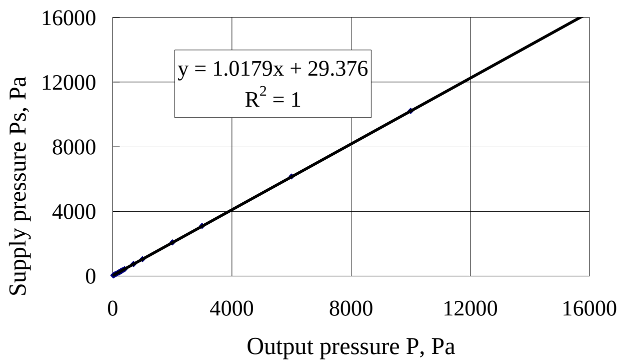

Fig.3. V1600 s/n 359. Dependence of actual supply pressure P_s of divider on output pressure P

The equation (5) written for the actual and for the ideal device is shown in tab. 1 (the measurements have been performed for the V1600, S/N: 359):

Table1

V1600 s/n 359. Real device	V1600 s/n 359. Ideal device
$P_s = 1{,}0179\, P + 29{,}376$	$P_s = 1{,}0179\, P + 28{,}967$

where: P_s = 29,376 and P_s = 28,967 – the pressure difference on the fixed throttle when is no piston on the nozzle, in the real and respectively ideal case. The actual value measured by calibrator was P_s = 28 Pa.
The equality of the angular coefficients means the pistons of the ideal and the actual device have identical displacement of the pistons with the change in pressure. Hence, the requirement of geometrical similarity is respected.

In order to calculate the ideal values of P_s, the software is using (3) and it is combining the ideal and the actual characteristics of the divider.

In the tab. 2 a discrepancy $\Delta P_s = (P_{sact} - P_{sid})$ between the ideal and the experimental values of the supply pressure P_s of the V1600 s/n 359 are shown:

Table 2

P_n, Pa	20	100	120	200	700
ΔP_s, Pa	0,1	-0,5	0,1	-0,6	0,3

Continuation of Table 2

P_n, Pa	1000	3000	6000	10000	16000
ΔP_s, Pa	4,3	21,1	21,9	10,7	-17,9

The actual value of the supply pressure P_s was measured by Druck calibrator DPI 145. The upper measuring limits of the two sensors are 0,5 bar and 1 bar and the uncertainty of the measured pressure (at 2 sigma level, determined by calibration) was less than 5 Pa.

Equation (5) for the actual device s/n 359 and for its mathematical model (10) is shown in tab.3:

Table 3

V1600 s/n 359. Real device	V1600 s/n 359. Ideal device
$P_s = 1{,}0179\, P + 29{,}376$	$P_s = 1{,}0179\, P + 39{,}892$

As it is visible from table 3 at an output pressure P=0 Pa, the pressure drop on the fixed throttle obtained by using the frictional model (10) is with 10,5 Pa more than measured.

For example the discrepancy $\Delta P_s = (P_{sac} - P_{sid})$ between the experimental values of supply pressure P_s of the V1600 s/n 359 and the values given by its mathematical model (10) are shown in tab.4:

Table 4

P_n, Pa	20	100	120	200	700
ΔP_s, Pa	16,4	3,2	1,7	-5,5	-26,0

Continuation of Table 4

P_n, Pa	1000	3000	6000	10000	16000
ΔP_s, Pa	-29,0	-29,5	-24,1	-11,6	8,0

Dependence of design Euler's criterion of two surveyed mathematical models on actual Euler's criterion of V1600 s/n 359 is shown on Fig. 4.

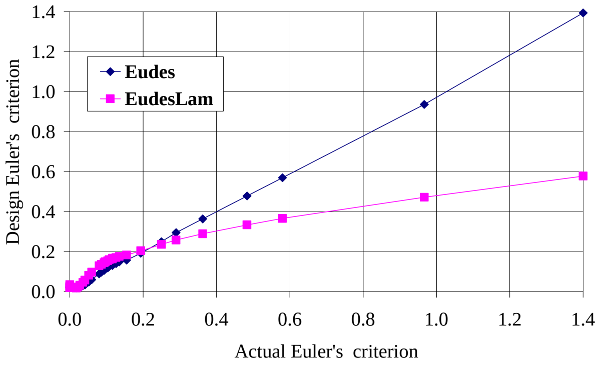

Fig. 4. V1600 s/n 359. The design Euler's criterion versus actual Euler's criterion

The graph presented in Fig. 4 suggests clearly that the gas streams, in the actual device and in its model based on Poiseuille's law, are dynamically similar. The mathematical model provide almost the same results with the real device when the losses of pressure are proportional to a quadrate of velocity.

In tab. 5, the values of the experimental and the design model (8) of Euler's criterion for V1600 s/n 359 are presented:

Table 5

P_n, Pa	20	100	120	200	700
Eudes	1,39	0,30	0,25	0,16	0,06
Euact	1,40	0,29	0,25	0,16	0,06

Continuation of Table 5

P_n, Pa	1000	3000	6000	10000	16000
Eudes	0,05	0,03	0,02	0,02	0,02
Euact	0,05	0,04	0,03	0,02	0,02

The V1600 s/n 359 was calibrated by comparison with the P7000 Pressurements standard (total uncertainty less then 50 ppm,) at the pressure P_c = 6 kPa. In the tab. 6 the mean value of the difference between effective area A calculated using the old and the new method of calibration, from the values A, obtained by direct comparison of the V1600 with the P7000 standard are shown in tab.6:

Table 6

P_n, kPa	$(A_{old} - A_{new})/ A_{new}$, %		σ, %
	Old method	New method	
8	0,006	0,003	0,0003
10	0,007	0,002	0,0006
12	0,008	0,001	0,0006
14	0,007	-0,002	0,0005
16	0,009	-0,002	0,0006

where: σ - standard deviation of the mean.

The data presented in the tab.6 was confirmed by the calibration results of many V1600 instruments, which have been carried out at Pressurements and VNIIMS.

The V1600 is rapidly gaining recognition and acceptation as a standard for low-pressure calibrations at various levels of the calibration hierarchy.

Conclusion

An important part of the calibration uncertainty of the effective area of the V1600 has been caused by a methodical error of measuring the pressure difference on the fixed throttle of the pneumatic transducer.

The new calibration approach is based on the equality of the Eulerian criterion of the actual device and its mathematical model for a quadratic dependence between the pressure difference on the fixed throttle and flow rate.

This new approach is permitting the use of the supply pressure provided by the mathematical model for the calibration of V1600's effective area.

This method allows an essential increase of the calibration accuracy of V1600 instrument within the range 3 kPa to 16 kPa.

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