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DETERMINATION OF THE EFFECTIVE AREA OF A GAS-PRESSURE BALANCE FOR LOW PRESSURES

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Abstract − This paper describes our attempts to determine the effective area of a pressure balance (Type V1600/4D, manufactured by Pressurements, Ltd.) that is used in our pressure-calibration laboratory as a standard pressure balance for gauge and differential pressure measurements. The gauge measuring range of this gas-operated balance extends from a nominal pressure of 20 Pa up to 16 kPa, and its minimum differential pressure measurement is 5 Pa. This pressure balance is a "discrete" system, and therefore it has several pistons and weights. In the measuring range from a nominal pressure of 400 Pa up to 16 kPa only two different pistons are used. From the calibration results relating to pressure and mass provided by the calibration laboratories of the NPL and Lm (Mass Laboratory of the MIRS) we derived an approximate formula for the effective area. This equation shows the non-linearity of the pistons' effective area. Good agreement between the indirectly measured and the approximated values of the effective area was obtained, and their deviations are more than four times smaller than the accuracy of this balance. Knowing the effective area of the piston will reduce the number of discrete calibration points that are required.

Keywords: pressure balance, effective area, gauge and differential pressure.

1. INTRODUCTION

Standard pressure balances of the type V1600/4D [1] are used in the Laboratory for Measurements in Process Engineering (LMPS) for low-pressure and differential pressure measurements in the measuring range from 20 Pa up to 16 kPa [2]. The pressure balance is a gas-operated balance that uses conical pistons, made from steel and aluminium, and a stainless-steel cylinder. The pressure is maintained by internal regulation of the gas flow through the piston and the cylinder. This is not a classical pressure balance because it operates as a "discrete" system and has several pistons and weights. In the measuring range from 400 Pa up to 16 kPa only two different pistons are used, and so in this range it could be treated like a classical pressure balance. As a result it is necessary to determine the effective area of the piston-cylinder assembly of the pressure balance [3-4]. The true pressure and the mass of the pistons and weights of our pressure balance were determined at the NPL and Lm laboratories respectively [5-6]. From these calibration results we derived an approximate formula for the effective area, which is also the main goal of this paper. The formula shows a non-linearity for this type of balance, and will be discussed in the paper. A knowledge of the effective area of the piston reduces the number of discrete calibration points.

2. THEORETICAL BACKGROUND

The operation of the V1600/4D pressure balance is based on the dynamic interaction of the gas flow with a non-cylindrical piston [1]. The main parts and a simplified model of this balance were presented in [2].

2.1. A mathematical model of the pressure balance

The effective area, A_{eff}, of the balance to be calibrated at the reference temperature of 20 °C is determined for each reference pressure, p_r, using the equation:

$$A_{\mathrm{effj}} = \frac{\left[\sum_{i=1}^{N} m_i g(1-\rho_a/\rho)\right]_j}{p_{rj}(1+\alpha(t_j - 20°\mathrm{C}))}, \quad (1)$$

where ρ_a is density of the air, ρ is the density of the weights, m is the mass of the pistons and weights, g is local acceleration due to gravity, α is the thermal coefficient of expansion for the piston and the cylinder, t is the temperature of the piston-cylinder assembly and suffix j denotes each calibration point [3]. From the analyses of the mean results for A_{eff}, three cases can arise [1]:

1. A_{eff} is constant;
2. A_{eff} is a linear function of the pressure

$$A_{\mathrm{eff}} = A_0(1+\lambda \cdot p_r), \quad (2)$$

3. A_{eff} is a quadratic function of the pressure

$$A_{\mathrm{eff}} = A_0(1+\lambda_1 \cdot p_r + \lambda_2 \cdot p_r^2), \quad (3)$$

where the constants λ_i are calculated using the least-squares method.

The basic equation that describes the operation of the balance, and which is given by the manufacturer [1], is:

$$p = p_{\mathrm{nom}} \frac{g_1}{g_n} \frac{1}{\alpha(t-20°\mathrm{C})}, \quad (4)$$

where g_n is the value of the pressure at the reference acceleration due to gravity at which the deadweight tester has been calibrated, and p_{nom} is the pressure at the nominal conditions.

3. DETERMINATION OF THE EFFECTIVE AREA

On the basis of the true pressure and the mass of the pistons and the cylinder of our pressure balance [5-6] the effective areas for two pistons were calculated according to (1). The obtained results, which are presented in figures 1 and 2, show the non-linearities of the pistons' effective areas. The best agreement between the indirectly measured (1) and the approximated values of the effective area were obtained if we used the formula:

$$A_{\mathrm{eff}} = A_0 (1 + \lambda \cdot (p_{\mathrm{nom}} - p_{\mathrm{lower}})^b) , \qquad (5)$$

where the constants λ and b are calculated from the least-squares method. Formula (5) gives better results than formula (3). The non-linearities of the effective area in the measuring range from 400 Pa up to 1000 Pa and from 1 kPa up to 16 kPa are shown in figures 1 and 2 respectively.

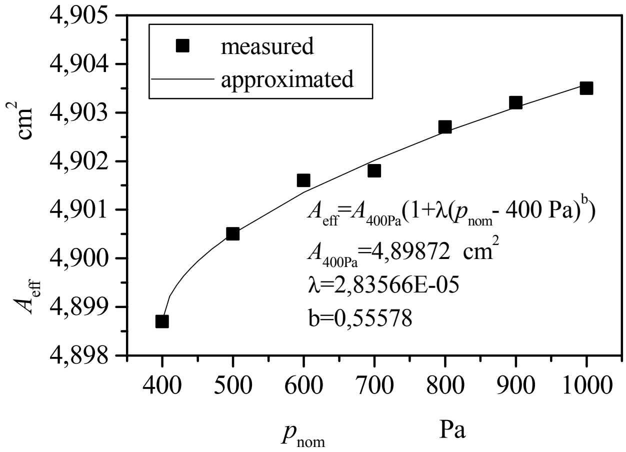

Fig. 1. Effective areas in the measuring range from 400 Pa to 1000 Pa

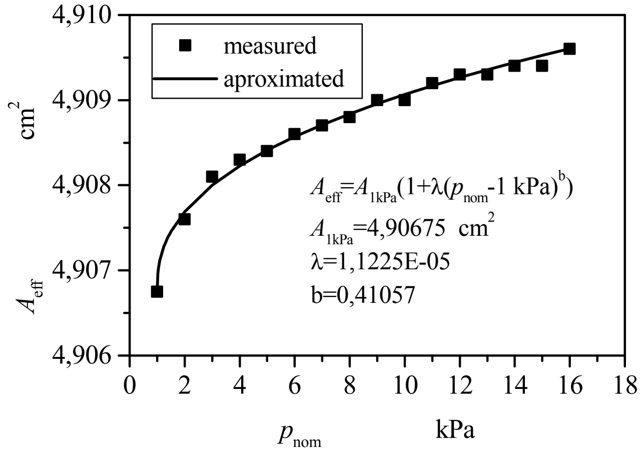

Fig. 2. Effective areas in the measuring range from 1 kPa to 16 kPa

The deviation between the indirectly measured (1) and the approximated values according to formula (5) for both measuring ranges were also calculated in Pa. In both measuring ranges the deviations were more than four times lower than the accuracy of the balance. The deviations for the measuring range from 400 Pa up to 1000 Pa and from 1 kPa up to 16 kPa are also shown in figures 3 and 4.

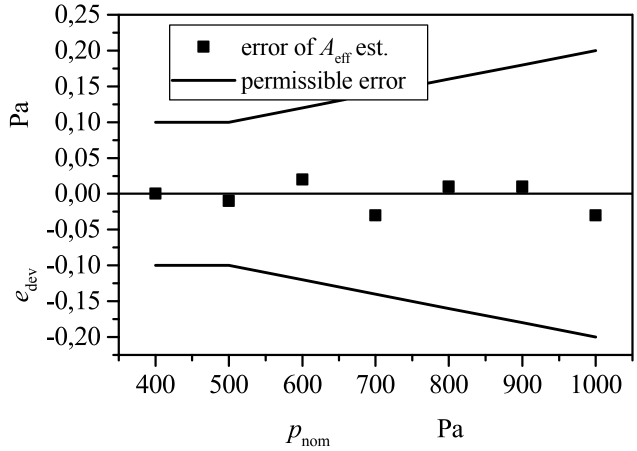

Fig. 3. Deviations between measured and approximated values of the effective area in the measuring range from 400 Pa to 1000 Pa

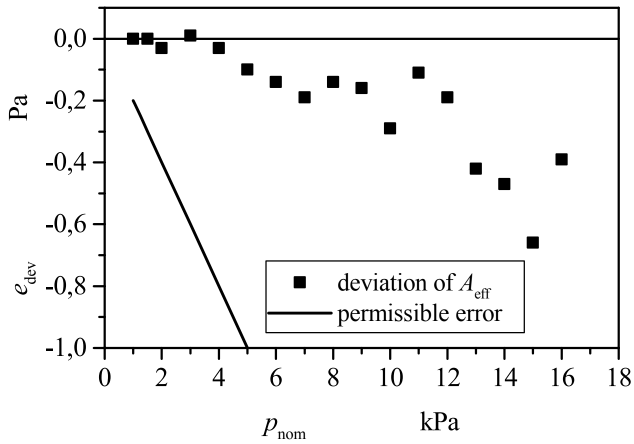

Fig. 4. Deviations between measured and approximated values of the effective area in the measuring range from 1 kPa to 16 kPa

The good agreement between the measured and approximated values also shows in the standard errors of the A_{eff} estimation, which is defined as:

$$SEE = \sqrt{\sum_{i=1}^{N} \frac{(A_{\mathrm{eff\,i}} - A_{\mathrm{eff\,(apr)i}})^2}{\text{N-M}}} , \qquad (6)$$

where A_{effi} is calculated according to (1), $A_{\mathrm{eff(apr)i}}$ is calculated according to (5), N is the number of measured points, and M is the number of parameters in (5). The obtained standard error of the A_{eff} estimation *SEE* is less than $\pm 1{,}6 \cdot 10^{-8}$ m^2 in the measuring range from 400 Pa up to 1000 Pa and less than $\pm 0{,}5 \cdot 10^{-8}$ m^2 in the measuring range from 1 kPa up to 16 kPa.

4. THE EVALUATION OF MEASUREMENT UNCERTAINTY AND RESULTS

The double standard-deviation method is used to express the measurement uncertainty [7]:

$$U(y) = 2s\,, \tag{7}$$

If a quantity y depends on the number of independent quantities x_i (i=1,N), the absolute uncertainty of the quantity y is:

$$U(y) = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial y}{\partial x_i}U(x_i)\right)^2}\,, \tag{8}$$

where x and y are the directly and indirectly measured quantities and $U(x_i)$ (i=1,N) and $U(y)$ are their uncertainties. The uncertainty of the quantity y can be expressed as the relative uncertainty $U_r(y)$:

$$U_r(y) = \frac{U(y)}{y} \tag{9}$$

On the basis of equations (1), (8), (9) and the standard error of the A_{eff} estimation (6), the relative uncertainty of the effective area of the piston-cylinder unit is:

$$U_r(A_{eff}) = \left[\left(\frac{t_{95,45;\nu}\cdot SEE}{A_{eff}}\right)^2 + \left(-\frac{U(p)}{p}\right)^2 + \left(\frac{\sum_{i=1}^{N}U(m_i)}{\sum_{i=1}^{N}m_i}\right)^2 + \left(\frac{U(g)}{g}\right)^2 + \left(\frac{-(t-20°C)U(\alpha)}{1+\alpha(t-20°C)}\right)^2 + \left(\frac{-\alpha\cdot U(t)}{1+\alpha(t-20°C)}\right)^2 + \left(\frac{-U(\rho_a)}{(1-\rho_a/\rho)\rho}\right)^2 + \left(\frac{\rho_a\cdot U(\rho)}{(1-\rho_a/\rho)\rho^2}\right)^2\right]^{1/2}, \tag{10}$$

where $t_{95,45;\nu}$ denotes the Student factor at the degree of freedom ν=N-M. The repeatability and the long-term stability of the pressure balance are not considered in (10).

If we want to determine the measurement uncertainty of the effective area of the piston-cylinder unit on the basis of (10), the uncertainties of the individual variables have to be known and evaluated. The values and evaluations are determined on the basis of the calibration certificates [5] and [6]. The uncertainties of the true pressure are determined according to [5]. On the basis of the piston mass and the weights mass data in the certificate [6], their uncertainties $U(m)$ are from ±0,1 mg to ±6.3 mg. This evaluation uses the data for densities, ρ_a = 1,2 kg/m^3 and ρ = 8000 kg/m^3, which are stated on the certificate. Their uncertainties are ±0,05 kg/m^3 and ±500 kg/m^3. The acceleration due to gravity to which the pressure balance has been calibrated is g_n = 9,80665 m/s^2, while the local acceleration due to gravity is g = (9,80630±10^{-5}) m/s^2. The thermal coefficient α is 0,000022 (°C)$^{-1}$ [6] and its uncertainty is estimated to be ±30 %. The temperature of the pistons and the weights during the calibration process was (20±2) °C.

On the basis of the described algorithm and the given magnitudes of the quantities and their uncertainties, the uncertainties of the effective areas were calculated for the whole measuring range, (10). The results are shown in figures 5 and 6. Figures 5 and 6 also show the permissible error limits, which are declared by the manufacturer [1].

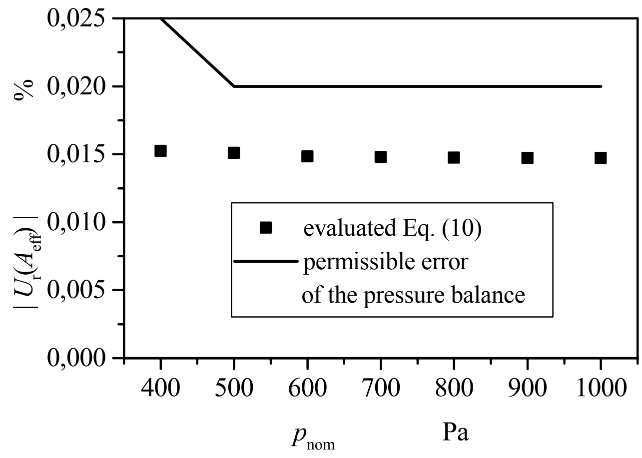

Fig. 5. The measurement uncertainty of the effective area in the measuring range from 400 Pa to 1000 Pa

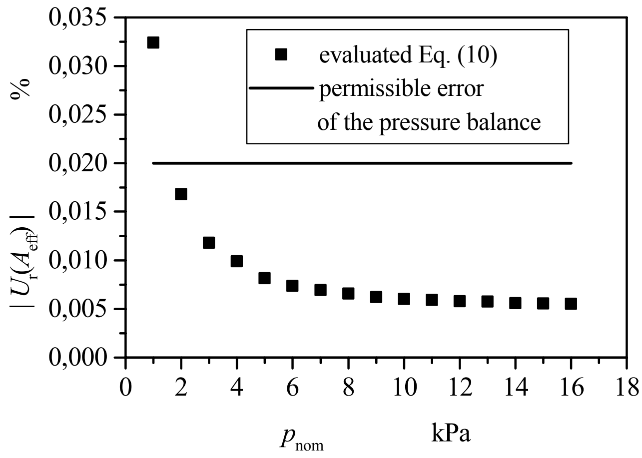

Fig. 6. The measurement uncertainty of the effective area in the measuring range from 1 kPa to 16 kPa

On the basis of the evaluation of the measurement uncertainty the largest contributions of the independent variables to the total uncertainty of the effective area $U(A_{eff})$ were made by the pressure p, the standard error of estimation *SEE*, the temperature of the pistons and weights t, and the mass of the pistons and weights m. In the measuring range from 400 up to 1000 Pa the contribution of the pressure is less than 55 % and the contribution of the *SEE* is less than 40 %. The largest influence on the relative error of the effective area in the measuring range from 1 kPa up to 16 kPa were made by the pressure p and the temperature of the pistons and weights t. Their contribution varied from 95 to 30 % and from 2 to 65 % respectively. These results are shown in figures 7 and 8.

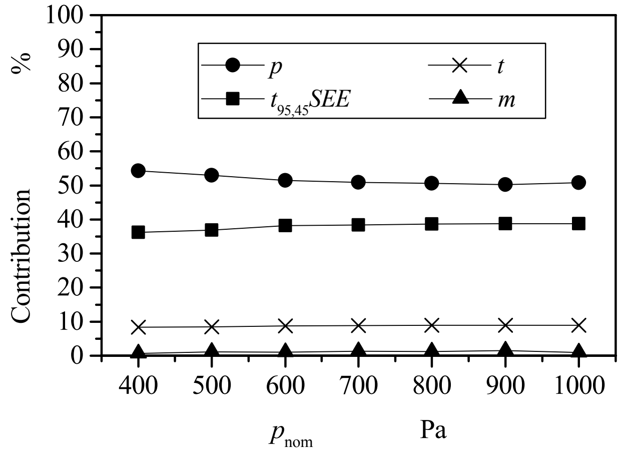

Fig. 7. Contribution to the total uncertainty of the effective area in the measuring range from 400 Pa to 1000 Pa

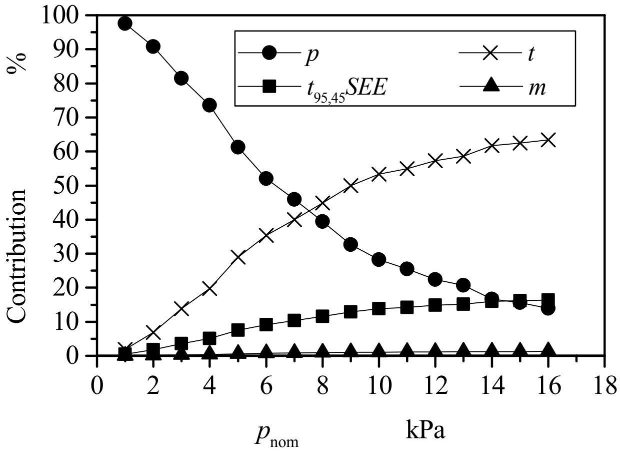

Fig. 8. Contribution to the total uncertainty of the effective area in the measuring range from 1 kPa to 16 kPa

5. CONCLUSIONS

We can conclude that the formula (5) is relatively good for fitting the non-linearity of our pressure balance. Knowing the effective area of such a pressure balance can reduce the number of discrete calibration points that are required. The evaluated measurement uncertainties of the effective area were less than the permissible error for the pressure balance in the whole measuring range from 400 Pa up to 16 kPa. The relative uncertainties of the A_{eff} do not exceed ±0,02 %. The most influential quantities when it comes to the total uncertainty of the effective area are the pressure, the standard error of the A_{eff} estimation, the temperature of the pistons and weights, and mass of the pistons and weights. The repeatability and the long-term stability were not considered in this analysis.

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