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ANALYSIS OF CAPACITANCE AND LINEARITY OF GAUGE CHARACTERISTIC OF COPLANAR MICRO-DISPLACEMENT SENSOR

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Abstract - Capacitive sensors can be applied for measuring different kind of non-electrical quantities, such as geometrical dimensions of subjects, thickness of films, displacement and vibration of grounded surfaces, position of the object and others.

Existing three-electrode micro-displacement sensors, made in the shape of flat capacitor geometry, do have high values of sensitivity, but they also have notable non-linear behaviour over their whole gauge characteristics.

A three-electrode coplanar sensor, subject of this paper, can be designed by calculation in such a way, that its gauge characteristic does have relatively large section with relative error of the non-linearity of its gauge characteristic equals 0,051 % within certain ratio of sensor dimensions.

The method of direct field-strength determination, in combination with the method of conformal mapping, is used for a precise analytical calculation of the sensor capacitance. Analysis of the capacitance and special cases of sensor design has been discussed.

Keywords: capacitance calculation, micro-displacement.

1. INTRODUCTION

The three-electrode capacitive sensor can be used for measuring different kind of non-electrical quantities, such as geometrical dimensions of subjects, thickness of films, displacement and vibration of grounded surfaces, position of object and others. Existing capacitive micro-displacement sensors made in the shape of flat capacitor geometry do have high values of sensitivity, but they also have a notable non-linear behaviour over their whole gauge characteristics.

The capacitive coplanar sensor for measuring micro-displacement has been calculated by Legoev [1], for the case that widths of electrodes are infinite. In practice, however the widths of the potential electrodes have finite dimensions, as shown in fig.1.

The purpose of this article is to analyse the partial capacitance C_{12} between the electrodes *1* and *2* of the sensor, shown on Fig.1, taking into account the finite width of both electrodes and to optimise the sensor design for practical dimensions. Also the linearity of the gauge characteristic of this sensor, in use as a linear micro-displacement sensor, will be investigated here.

2. ANALYSIS OF THE CAPACITANCE OF COPLANAR MICRO-DISPLACEMENT SENSOR

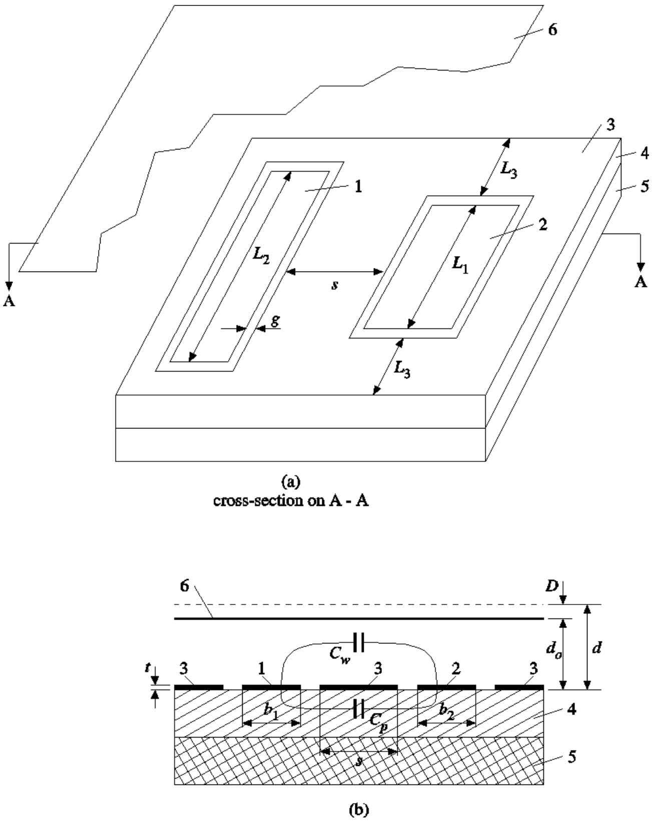

Fig.1. Linear coplanar micro-displacement sensor: (a)-structure of the sensor; (b)-cross-section of sensor; 1,2-high and low potential electrodes; 3-guard; 4 -dielectric material; 5-metal plate; 6-the surface of grounded conductive object.

The photolithography method can be effectively applied in sensor, shown on Fig.1 where high potential **1**, low potential **2** and guard **3** electrodes are fulfilled by exposing the metal film on dielectric material **4**, fixed on grounded metal plate **5**.

The analytical method of direct field-strength determination, applied for calculation of a flat-parallel field [2], in combination with the method of conformal mapping can be elegantly used to calculate the precise capacitance of the micro-displacement sensor, shown on Fig. 1.The gaps g between the electrodes **1**and **2** and the surrounding shield electrode **3** are assumed to be zero. The length L_2 of electrode **1**, acting as non-zero potential electrode, must be equal $L_2 = L_1 + 2L_3$. In that case, the potential field in the environment of electrode **2** is perfectly two-dimensional and each cross section perpendicular to the length L_1 will have identical shape, satisfying the flat-parallel field distribution condition.

For capacitance calculation it is necessary to assume the inside part of the calculated model, presented in Fig. 2 (a) as a cross-section of the sensor, as a part of the complex variable Z-plane, shown in Fig. 2(b). The following step is to map this part of the complex Z-plane into the upper half-part of the complex ς -plane, given in Fig. 2(c). In this case the point A_0 being the origin in the complex Z-plane, but also being a point in electrode **3** halfway the distance ***s*** between electrodes **1** and **2**, is mapped into the origin A' with coordinates $\xi = a_o$=0, η=0, of the complex ζ -plane. The point $A_4 = jd$ in the complex Z-plane is mapped into respectively $A_4^{'}$ with coordinates $\xi = a_4 = \infty$, η=0 and $A_4^{'}$ with coordinates u =-a_4=-∞, η=0 in the complex ζ -plane. All the other points $A_1, A_2, A_3, A_5, A_6, A_7$ located according to Figure. 2.b, are mapped into the points $A_1^{'}, A_2^{'}, A_3^{'}, A_5^{'}, A_6^{'}$ and $A_7^{'}$ respectively. This conformal mapping is carried out with the mapping function [2]:

$$\zeta = d\,\mathrm{th}\frac{\pi z}{2d}. \tag{1}$$

Having established the mutual relationship between the points in the complex Z- and ζ -planes, with the help of (1) the following equations can be found:

$$a_1 = d\,\mathrm{th}\frac{\pi s}{4d}\ ; \tag{2}$$

$$a_2 = d\,\mathrm{th}\frac{\pi\left(s+2b_2\right)}{4d}\ ; \tag{3}$$

$$a_3 = d\ ; \tag{4}$$

$$a_5 = d\,\mathrm{th}\frac{\pi\left(s+2b_1\right)}{4d}. \tag{5}$$

The value for the modulus of the field strength $E_{\eta=0}$ on the boundary $\eta = 0$, the absolute value of the charge τ_2 distributed over low voltage electrode **2** (which is at zero potential) and the absolute potential difference ΔV_{12} between electrodes **1** and **2** are determined by the following equations:

$$E_{\eta=0}\left(\xi\right) = \frac{B}{\left|\xi + a_1\right|\left|\xi + a_5\right|}; \tag{6}$$

$$\tau_2 = \varepsilon\int_{a_1}^{a_2} E_{\eta=0}\left(\xi\right)\partial\xi = \varepsilon B\int_{a_1}^{a_2}\frac{\partial\xi}{a_1a_5+\left(a_1+a_5\right)\xi+\xi^2} = \frac{\varepsilon B}{a_5-a_1}\ln\frac{\left(a_1+a_2\right)\left(a_1+a_5\right)}{\left(a_2+a_5\right)2a_1}; \tag{7}$$

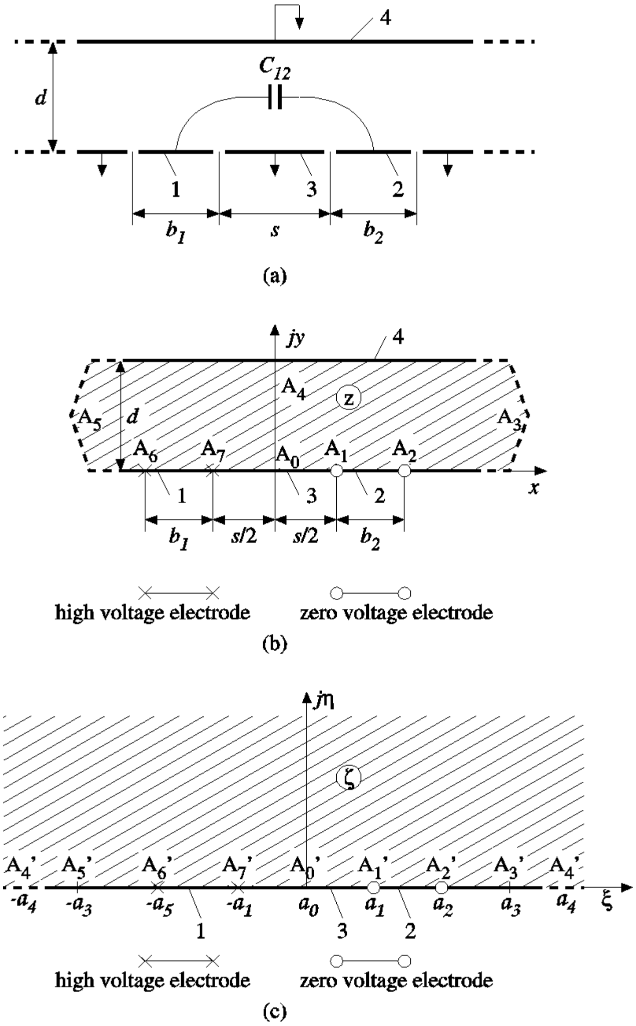

Figure 2.Coplanar capacitive micro-displacement sensor: (a)-cross section of the sensor; (b)-calculated model of the sensor; (c)-the system of electrodes lying in mapped plane.

$$\Delta V_{12} = \pi\lim_{\xi\to -a_1}\left(\left[\xi-\left(-a_1\right)\right]E_{\eta=0}\left(\xi\right)\right) = \frac{\pi B}{a_5-a_1}, \tag{8}$$

where $\varepsilon = \varepsilon_0\varepsilon_r$ is the permittivity of the concerning medium ; *B* is a constant ; a_1, a_2, a_5 are coordinates of electrode's boundaries in mapped plane.

The partial capacitance C_{12} between the electrodes **1** and **2** of the sensor (per unit of length) is calculated by the expression:

$$C_{12} = \frac{\tau_k}{\Delta V_{12}} = \frac{\varepsilon_0\varepsilon_r}{\pi}\ln\frac{\left(a_1+a_2\right)\left(a_1+a_5\right)}{2a_1\left(a_2+a_5\right)}, \tag{9}$$

Substitution of the equations (2) -(5) yields:

$$C_{12}=\frac{\varepsilon_0\varepsilon_r}{\pi}\ln\frac{\left(\operatorname{th}\frac{\pi s}{4d}+\operatorname{th}\frac{\pi(s+2b_2)}{4d}\right)\left(\operatorname{th}\frac{\pi s}{4d}+\operatorname{th}\frac{\pi(s+2b_1)}{4d}\right)}{2\operatorname{th}\frac{\pi s}{4d}\left(\operatorname{th}\frac{\pi(s+2b_2)}{4d}+\operatorname{th}\frac{\pi(s+2b_1)}{4d}\right)}.$$

This equation is identical to the expression, found by Heerens [3], which was calculated by the method of separation of variables.

From this formula a number of specific cases can be derived. The first three are quite obvious, where (a)-the distance *d* is infinite small (*d*=0); (b)-the width of electrodes are infinite small ($b_1 = b_2 = 0$);(c)-the extreme wide guard electrode (*s*=∞), resulting for the partial capacitance C_{12} into:

$$C_{12}\big|_{d=0} = C_{12}\big|_{b_1=b_2=0} = C_{12}\big|_{s=\infty} = 0 \quad . \qquad (10)$$

A next special geometry is obtained when the guard is absent (*s*=0), leading for the partial capacitance C_{12} to:

$$C_{12}\big|_{s=0} = \infty \, . \qquad (11)$$

In sensor design, if the total width $(b_1 + b_2 + s)$ is constant, the choice of equal width $(b_1 = b_2 = b)$ of electrodes leads to an optimal value for C_{12} . In this case the capacitance is found by the formula:

$$C_{12}\big|_{b_1=b_2=b} = \frac{\varepsilon}{\pi}\ln\frac{\left(\operatorname{th}\frac{\pi s}{4d}+\operatorname{th}\frac{\pi(s+2b)}{4d}\right)^2}{4\operatorname{th}\frac{\pi(s+2b)}{4d}\operatorname{th}\frac{\pi s}{4d}} \, . \qquad (12)$$

For the geometry with finite electrode dimensions the asymptotic expansion for large distance *d* ($d \to \infty$) leads to a constant value for C_{12} , given by:

$$C_{12}\big|_{d=\infty} = \frac{\varepsilon}{\pi}\ln\frac{(s+b_1)(s+b_2)}{s(s+b_1+b_2)} \, . \qquad (13)$$

Another special case is found if the widths of both potential electrodes are infinite ($b_1 = b_2 = \infty$), resulting into:

$$C_{12}\big|_{b_1=b_2=\infty} = -\frac{\varepsilon}{\pi}\ln\left(1-e^{-\pi s/d}\right) . \qquad (14)$$

It can be noticed that this equation is the same as the expression, found by Legoev [1], in the case of infinitely wide electrodes.

Capacitance values calculated with the help of (14) have been shown in Table 1 and plotted in Fig.3 as a function of *d/s* for various values of *b/s*.

It can be seen, that the curves exhibit a non-linear behaviour in general. But if the ratio *b/s* is increased then the gauge characteristic of the sensor becomes closer to a linear one.

3. THE LINEARITY CALCULATION OF THE SENSOR GAUGE CHARACTERISTIC.

The nominal gauge characteristic of the sensor is determined for the following conditions:

- the widths **m** of the potential electrodes is equal infinity (m=∞);

TABLE 1. Partial capacitance C_{12} per unit of length (pF/m), calculated for various values of ratios d/s respectively b/s.

d/s	b/s=1	b/s=2	b/s=3	b/s=4	b/s=5	b/s=6	b/s=∞
1,0	0,114	0,124	0,124	0,125	0,125	0,125	0,125
1,2	0,185	0,211	0,213	0,213	0,213	0,213	0,213
1,4	0,256	0,309	0,315	0,316	0,316	0,316	0,316
1,6	0,322	0,411	0,424	0,426	0,426	0,426	0,426
1,8	0,381	0,511	0,536	0,540	0,541	0,541	0,540
2,0	0,433	0,607	0,646	0,655	0,656	0,657	0,657
2,2	0,478	0,697	0,754	0,768	0,772	0,772	0,773
2,4	0,516	0,779	0,858	0,879	0,885	0,887	0,887

-the parasitic capacitance C_p between high- and low potential electrodes (Fig.1) is equal to zero (C_p =0);

-the thickness of metal plate **t** and the gap between electrodes **s** are equal to zero (t=s=0);

-the length L_3 of the guard and L_2 of the high potential electrodes are equal infinity ($L_3 = L_2 = \infty$);

-the lack of roughness and non-flatness of electrode's surfaces is required;

- the relative permittivity of air is ε_r =1,00056.

When ratio *b/s* ≥6, the gauge characteristic, due to changing of the parameter *b,* shows almost linear behaviour. In this way, this type of sensor can be used as linear micro-displacement sensor. In order to find the boundaries of the most linear part of the characteristic, it is worthwhile to investigate in more detail the function of $C_{12} = f(d/s)$, in order to find the boundaries of the most linear part of the characteristic.

The function, given by (14) is closest to linear in the inflection point $d = d_0$, where the second derivative equals to zero:

$$C_{12}''\left(\frac{d_0}{s}\right)=-\varepsilon\pi\left[\left(\frac{s}{d_0}\right)^3\frac{1}{e^{\pi s/d_0}-1}\left(\frac{\pi e^{\pi s/d_0}}{d_0/s}\frac{1}{e^{\pi s/d_0}-1}-2\right)\right]=0.$$

Having solved this relation numerically the inflection point is found at ratio d_0/s =1,971351.

The tangent line through the inflection point can be used as a practical gauge characteristic for the displacement sensor. In that case the expression for the characteristic with linear dependence of its capacitance C_{lin} from displacement ***d*** of plate 6 (Fig.1) is given by:

$$C_{lin} = C_0 + S\frac{D}{s}, \qquad (15)$$

where C_0 is nominal value of the capacitance, while ($d = d_0$ or D=0); S is absolute sensitivity of the sensor; $D = d - d_0$ is the range of the displacement.

Substitution of the real non-linear gauge characteristic of the sensor (14) for the linear one (15) leads to relative error $\Delta d / D$ of displacement measurement, which can be found by formula

$$\gamma_{ne} = \Delta d / D = \left|\frac{C_{lin} - C}{DS}\right|, \qquad (16)$$

where Δd is the absolute error of displacement measurement.

If all equations in (16), having $d = d_0 + D = d_0(1 + D/d_0)$ as a parameter are developed in expansion of at least up to the 3-d order, while $D/d_0 << 1$, then the simple expression for calculating the error of the non-linearity of the gauge characteristic of micro-displacement sensor can be approximated by:

$$\gamma_n = \Delta d / D \approx 0{,}198\left(D/d_0\right)^2 \approx 0{,}051\left(D/s\right)^2. \qquad (17)$$

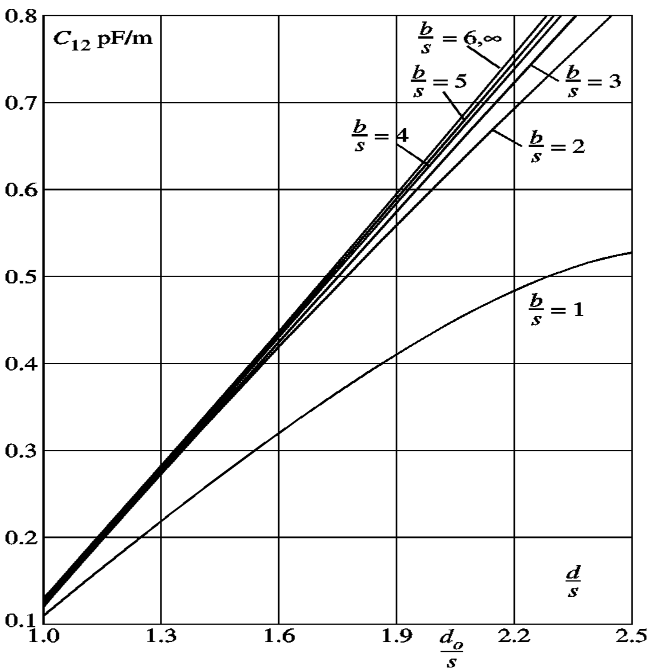

Fig.3.The partial capacitance C_{12} calculated per unit of length as a function of relative distance *d/s* for various values *b/s*.

Values of approximated non-linearity error γ_n and the exact one γ_{ne}, calculated by equations (17) and (16) respectively, have been listed in Table 2.

TABLE 2. Values of the non-linearity error, calculated by exact (16) and approximated formulas (17), respectively.

D/s	-0,2	-0,1	0	0,1	0,2
γ_n ,%	0,204	0,051	0	0,051	0,204
γ_{ne} ,%	0,228	0,054	0	0,048	0,183

4.CONCLUSIONS

It has been shown that the analytical method of direct field-strength determination can be applied to calculate and analyse the precise capacitance of three-electrode coplanar micro-displacement sensor.

This type of sensor can be used for linear micro-displacement measurement of grounded surfaces, geometrical dimensions of subjects, thickness of films, vibration of grounded surfaces, position of the object and others.

The received results allow the optimising of the sensor design and archive the relative error of the displacement in second order. For example, when the ratio of D (displaced subject from nominal distance d_0) to dimension s (the width of guard electrode) D/s = ± 0,1,the value of non-linearity of sensor gauge characteristic is γ_n =0,051%.

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