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QUATERNION DATA ANALYSIS OF THREE AXIS ACCELERATION MEASURED WITH NEWLY DEVELOPED SMALL SENSOR BALL

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Abstract – In order to show a concrete application of the signal processing based on quaternion expression, a prototype of sealed measurement module which could be thrown into the stream of liquid or powdery materials was developed. This module's shape is a small ball of about 40 mm across in diameter. And it consists of low power one-chip micro-controller, a couple of dual axis accelerometer, EEPROM, and batteries. So that, it can measure and record three dimensional acceleration by itself. As a preliminary experiment, the small sensor ball were suspended with a thick cord and swung by the speed controlled motor. Then, the data recorded under such condition were analyzed by using the spectra upon a form of quaternion. Through the experiments and observations, we could confirm that the small sensor ball could acquire data successfully. And, the relationships between the trend of the ball's motion and recorded data were considered under the present test condition. If the weight and measuring range are improved in futer, this sensor ball system will grow to an evaluating system of a state of stream in a closed container such as mixing or churning device.

Keywords : three axis acceleration, quaternion, stand-alone sensor

1. INTRODUCTION

We had proposed a concept to deal triple axis measurement signals as imaginary parts of single quaternion signal, and led the transformation which takes a quaternion signal into a sum of periodic signals as Fourier transformation [1-2].

On the other hand, high performance low power one-chip micro-controller and acceleration sensor made by micro machining technology have become popular. Nowadays they can be purchased at low price. So, using those commercial devices, the small sensor ball to measure three dimensional acceleration could be constructed easily.

Linking those situations, and giving further the battery powered activity and waterproof performance to such sensor ball, the special device which has a function of memorizing an acceleration history acted on it in the stream can be realized. Then analyzing the time series data obtained from such device by our concept of signal processing with quaternion, a new measurement system to evaluate the mixing performance or trend of churned material's stream in a closed container may be established. In this paper, prototype smal sensor ball developed for considering the possibility of such measurement system and the experiment to check its basic measuring function are described.

2. SMALL SENSOR BALL

The specification and the outline of the structure about the prototype sensor ball are shown in Table 1 and Fig.1. The main-circuit and rechargeable batteries of the sensor ball were put into the inside of a table tennis ball. A couple of dual axis acceleration sensor chips were fixed with a right angle to each other inside the ball. The space around the main-circuit was filled with silicone sealant. The interface to communicate with the micro-controller is a EIA232, and thin cable is used to connect between the sensor ball and a PC (personal computer).

A procedure of the sensor ball usage is;

0) Transferring the programs and setting parameters into the sensor ball from PC with the cable connection.
1) Charging the batteries.
2) Reset and activating the micro-controller of the ball.
3) Setting the sensor ball under the condition to measure.
4) Triple axis acceleration measurement starts by timer and the data were stored with EEPROM.
5) Picking up the sensor ball, and linking to a PC.
6) Analysis of the recorded data will be done on the PC.

TABLE 1 Specification of the prototype small sensor ball

Object	Acceleration (X,Y,Z)
Range	-19.6m/s^2 to $+19.6$m/s^2
Low Pass Filter	$f_c = 50$ Hz
Sampling Frequency	134 Hz (8bit, 3ch)
Resolution, Noise	0.17m/s^2, 0.018m/s^2(rms)
Recording time	5.5s (x 65 times)
Size, Exterior shape	Dia. 40mm, a sphere
Power supply	5V (3.0V battery x 2)

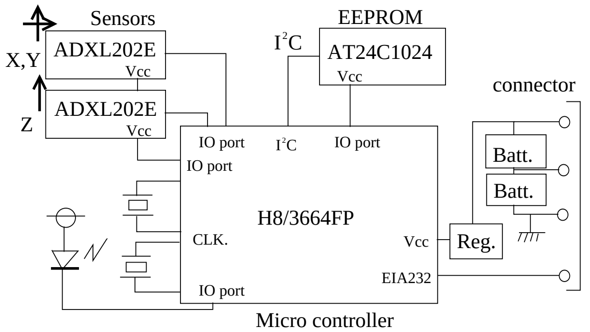

Fig. 1 Diagram of the prototype small sensor ball

3. DATA PROCESSING

Quaternion form "$\boldsymbol{q}_s(t)$" of triple axis acceleration data "$a_x(t), a_y(t), a_z(t)$" is defined as next;

$$\boldsymbol{q}_s(t) = a_x(t)\boldsymbol{i} + a_y(t)\boldsymbol{j} + a_z(t)\boldsymbol{k} \tag{1}$$

here, $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are the units stand for the imaginary part of Quaternion. A real part of $\boldsymbol{q}_s(t)$ is set to zero value. The transformation which divide the signal $\boldsymbol{q}_s(t)$ into the sum of periodic time function $\boldsymbol{Q}_s(f)\exp(2\pi f t \boldsymbol{P})$ is defined as follows;

$$\boldsymbol{Q}_s(f) = \frac{1}{N}\sum_{t=0}^{(N-1)/f_s} \boldsymbol{q}_s(t)\exp(-2\pi f t \boldsymbol{P}) \tag{2}$$

here, $\boldsymbol{Q}_s(f)$ is quaternion coefficient at frequency f, and $\boldsymbol{P}$ is constant vector quaternion whose norm is 1. Since (2) is a discrete formula, "t, f" are discrete values as follows; $t = n/f_s\,(n = 0, ..., N-1)$, $f = \pm m f_s/N\,(m = 0, ..., N/2-1)$. N means the number of the data. f_s means the sampling frequency. And "exp" function is defined as next;

$$\exp(a\boldsymbol{P} + r) = r\{\cos(a) + \boldsymbol{P}\sin(a)\} \tag{3}$$

here, a and r are real numbers. We consider the coefficients calculated with (2) as a quaternion spectrum. The basic characteristics and rules of arithmetic operations on quaternion are mentioned in [3].

4. EXPERIMENT

A preliminary experiment was carried out in order to check the basic functions of the sensor ball.

4.1 Apparatus

Construction of the apparatus for experiment is shown in Fig.2. Prototype sensor ball was suspended and located in the paper cylinder whose diameter is lager than the sensor ball, and wasn't fixed to the cylinder, so that, the sensor ball can move around within the cylinder. The paper cylinder was fixed to the shaft of the motor. The rotating speed of the motor was controlled to keep a constant velocity by the motor controller. The wire to hang is thick cable and indicate flexibility, but is hard to twist. Using this apparatus, we could give the sensor ball the periodic accelerations of which direction are varying in time.

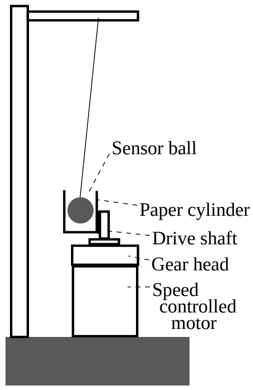

Fig. 2 Apparatus

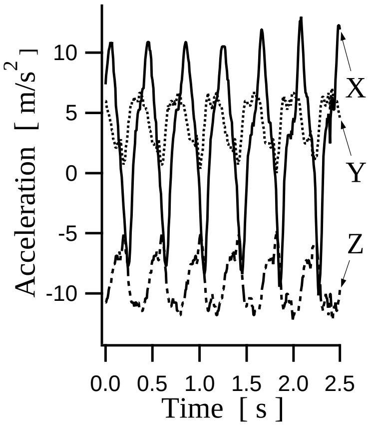

Fig. 3 A raw data at 440 rpm

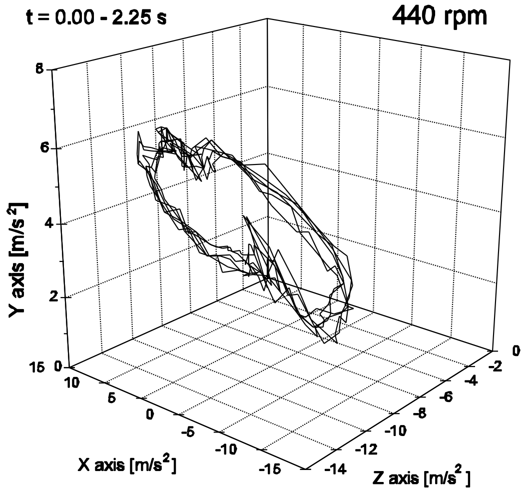

Fig. 4 Time trace of $\boldsymbol{q}_s(t)$

4.2 Experimental condition

The length between the hanging pivot to the sensor ball is 0.60m. The gear ratio of the gear head is 3 : 1. The radius of the motor's drive shaft is 6.0mm, and the diameter of the paper cylinder is about 51.0mm. Considering the thickness of the paper material, the sensor ball can move in a cylindrical space which radius is about 58.0mm at maximum. The sampling frequency is 134.0Hz, and the value of the N is 670, so that the recording time is 5.0s long at each measuring. For the calibration of the sensor system, gravity at rest was measured.

4.3 Observation and Acquired DATA

The motor speed setting was changed from 100 rpm to 520 rpm by 40 rpm step in most cases. As the result, according to the rotating speed, two trends of the movement of the sensor ball were observed. One is the single circular orbit occurred at high speed, which is comparatively smooth but involve some skip motion. Another one is dual motion synthesized from large circular orbit around the motor shaft and small circular orbit within paper cylinder. It occurred at the low speed less than 160 rpm in this experiment. A transient state of motion also observed at around the 160 rpm. The time trace of ac-

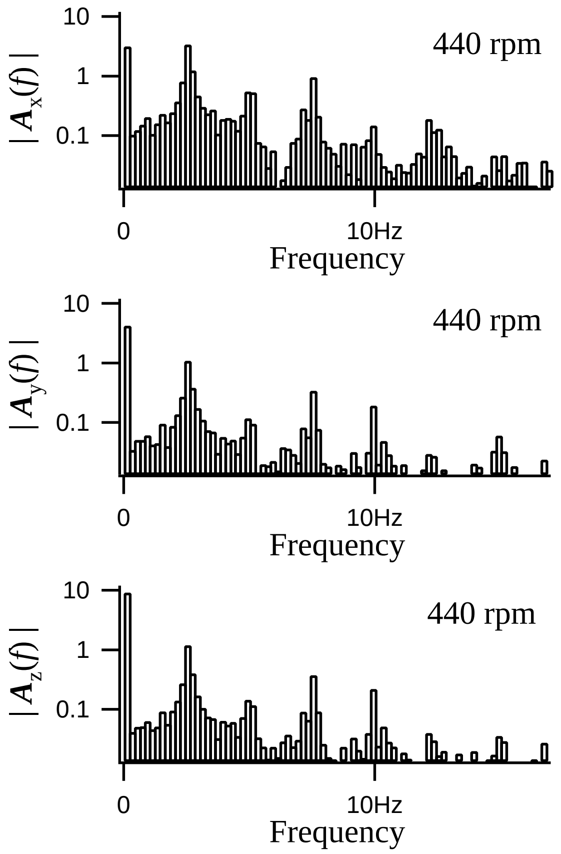

Fig. 5 An example of $|\boldsymbol{A}_x(f)|$, $|\boldsymbol{A}_y(f)|$, $|\boldsymbol{A}_z(f)|$

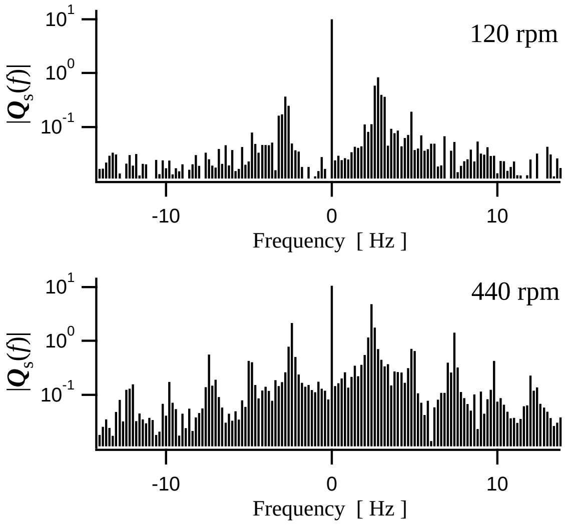

Fig. 6 The examples of the spectra $|\boldsymbol{Q}_s(f)|$

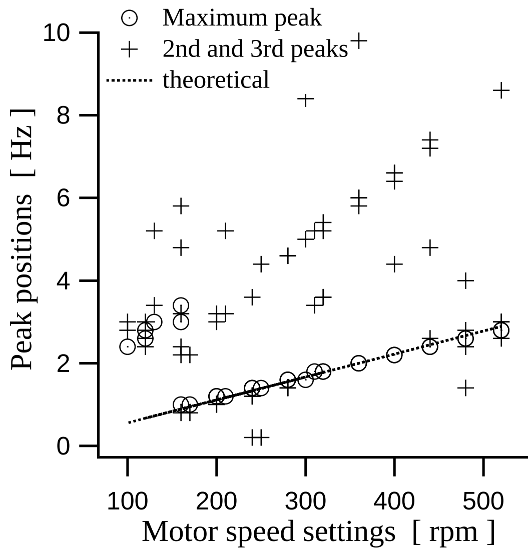

Fig. 7 Peak positions on the resultant spectra

celeration at 440 rpm measured by the sensor ball is shown in Fig.4. Under the condition as above, measurements and data processing were carried out repeatedly. Then, the spectra which indicate the peaks corresponding to the characteristics motion were given lastly. Positions of the main peaks in spectra were in agreement with the frequency calculated from motor speed (it's mentioned later). As an example of the analysis, the spectra of each axis $|\boldsymbol{A}_x(f)|$, $|\boldsymbol{A}_y(f)|$, $|\boldsymbol{A}_z(f)|$ are shown in Fig.5, and the spectra of amplitude $|\boldsymbol{Q}_s(f)|$ at 120 rpm and 440 rpm are shown in Fig.6. There are some peaks about the motion around the motor shaft and about the local cyclic motion in the paper cylinder. To calculate the spectra, the value of $\boldsymbol{P}$ was set to the direction of the gravity when the sensor ball is at rest.

5. DISCUSSIONS

Certainly, periodic change of acceleration could be found from each channels in the raw data or the spectrum of a scalar signal of each channels, but it is too hard to understand the three dimensional motion in mind. Comparing with this, the spectra with quaternion make it more easy, because of "$\boldsymbol{Q}_s(f)\exp(2\pi ft\boldsymbol{P})$" means an ellipse orbit in the $\boldsymbol{ijk}$ space. So you can grasp the image of the periodic component of the measured data from the sensor ball system with "Amplitude; $\boldsymbol{Q}_s(f)+\boldsymbol{Q}_s(-f)$, $\boldsymbol{Q}_s(f)-\boldsymbol{Q}_s(-f)$" and "Phase; Attitude and time shifting" of the elliptic motions.

5.1 Motor speeds and locations of the peaks

As a basic matter, a relationship between motor speed settings and the peak positions of the spectra were examined. At first, a next characteristic was observed. Though the peak values were different from each other, most peaks appear at corresponding position in both of positive and negative frequencies. The positions of main three peaks were picked up at every experiments, and the result is shown in Fig.7 for positive frequency.

Since the directions of gravity vector is stable, main periodical changes of acceleration vector which appear on measurement data depend on motor speed settings. Main component's frequency of the data calculated from motor speed setting with gear ratio is indicated with a broken line in Fig.7. Experimental main peaks was in agreement with the theoretical values in the range above 160rpm. The reason why main peak positions differ from the theoretical values in the range

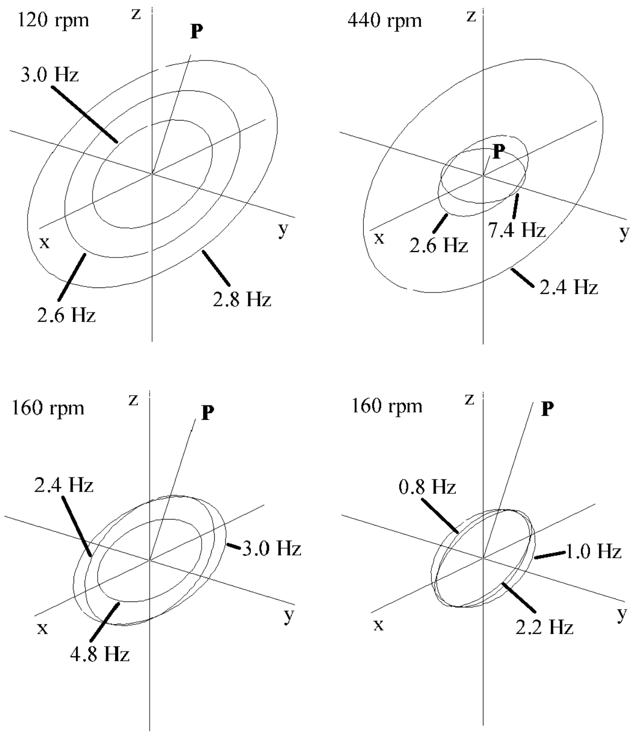

Fig. 8 Displaying of major peaks' components

under 160rpm may be that local cyclic motion within paper cylinder would be larger than comparatively slow motion of rotating around the motor shaft. From this coincidence, it was thought that the prototype sensor ball could acquire data successfully.

5.2 Graphic display of the elliptic orbit

Triple axis measurement signals can be divided into elliptic orbit component in three dimensional space in general. Because of any frequency component synthesized with a couple of positive frequency coefficient and negative one become to ellipse orbit in 3D space as next;

$$\boldsymbol{Q}_s(f)\exp(2\pi f t\boldsymbol{P}) + \boldsymbol{Q}_s(-f)\exp(-2\pi f t\boldsymbol{P}) = \boldsymbol{A}\cos(2\pi f t) + \boldsymbol{B}\sin(2\pi f t) \quad (4)$$

Here, $\boldsymbol{A}$ and $\boldsymbol{B}$ are vector quaternion as log as the input measurement signals are triple axis. And, independently of how $\boldsymbol{P}$ is set, $\boldsymbol{A}$ and $\boldsymbol{B}$ become the constant values to the same signal. Displaying the major components of the measurement data with the ovals like as shown in Fig.8 are very helpful to grasp the characteristics.

5.3 Effect of the sensor's attitude varying

The prototype sensor ball has no angular rate sensors, so that, we can't know the direction of sensor axes at field measuring. However, since the sensor ball's shape is spherical and small, little unbalance of surface condition or flow condition which cause rotating torque to the sensor body would be exist. Furthermore, a high speed rotation encounters damping force in the fluid. Therefore, the change of the sensor's attitude might be comparatively slow and not so much effective to statistical observation for the flow condition.

5.4 Other points

Two problems were also considered about this prototype sensor ball. One is the density. The specific gravity of the prototype sensor ball is much heavier than typical fluid, so the device will soon sink when it is thrown in the liquid. Another one is the output saturation when the sensor ball is acted by severe shock. When the sensor ball is thrown into the mixing device actually, those points may bring undesirable results.

Since it is assembled from electrical parts of semiconductors, especially the batteries are required to activate by standalone, it is too hard to make it more low density. However, if it is possible to make the sensor ball smaller, average density as a whole can be decreased by over-coating with low density materials like styrene foam.

To solve the output saturation problem, other less sensitive sensor chip can be adopted. For instance, $\pm 98.0\text{m/s}^2$ type is available in same sensor chip series. And an over-coating is also profitable mean to make less sensitive in high frequency range at real measuring.

6. CONCLUSION

The prototype small sensor ball is simple and not so much high performance. However, through the observations and considerations about the experiments, we could confirm that the small sensor ball could acquire data successfully, and the acceleration histories acted on it can be analyzed by dividing the data into the periodic motions. So that, when the properties of the attitude change of the sensing axes are predictable during a measurement, it could be possible to grasp the major trends of the small sensor ball's motion easily.

After this, improving the weight of the sensor ball and measuring range and giving more durability, we expect that this measurement system will grow to an evaluation system of a state of the stream in a closed container such as mixing or churning device.

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