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OPTICAL METHOD OF CAUSTICS - FULFILLED EXPERIMENTAL APPLICATION TO THE CONTACT PROBLEM

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Abstract – Experimental optical method of caustics is established for solving the singularity at the crack tip through the stress intensity factor evaluation. Method is advantageously improved in application to mechanically anisotropic materials such as fiber-reinforced composites. Recently, the experiments are performed for different types of isotropic body contacts and the analysis of optical effects. Task of experiments is prediction of the future of inspected material surface and the assessment of structure damage. Theoretical fundamentals will prospectively provide the application of the method of caustics to the contact of the structural parts made of composites.

Keywords: mechanics, caustics, contact

1. INTRODUCTION

Optical method of caustics is commonly in use for the experimental determination of the stress intensity factor *K* either in transparent or non-transparent light arrangement [1]. The concentrated field of light in the zone surrounding the crack tip generates the optical effect - dark spot called caustics, surrounded by the concentrated light on its edge. The shape of caustic curve is of the same epicyclical form for any kind of isotropic material for the crack I opening mode (Fig. 1).

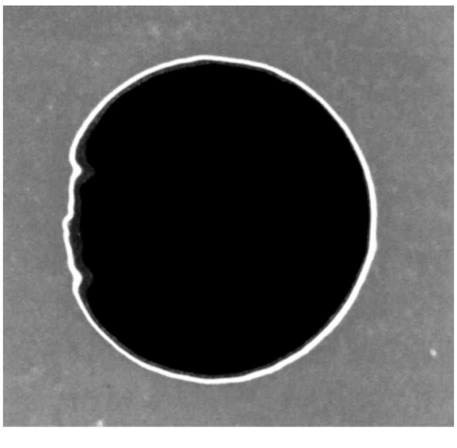


Fig.1. Mode I caustics around the crack tip

The invented method applied to the non-transparent mechanically anisotropic materials enables the determination of stress intensity factors measurements at different types of fiber-reinforced composite materials. The solution cannot be given in the general form and should be considered separately for each problem.

2. THEORY OF CONTACT CAUSTICS

As the singularity at the crack tip is analyzed by the method of caustics, similarly is done by the contact force between two elastic bodies [2]. Solution is given for the force *F* that is inclined by the angle α to the plate (Fig. 2):

$$\sigma_r = \frac{2F}{\pi r}\cos(\alpha + \varphi), \quad (1)$$

where the components σ_φ and $\tau_{r\varphi}$ are equal to zero. Boundary conditions are satisfied at the edge of the plate, except at the contact point ($r = 0$), where the stress σ_r goes to infinite value.

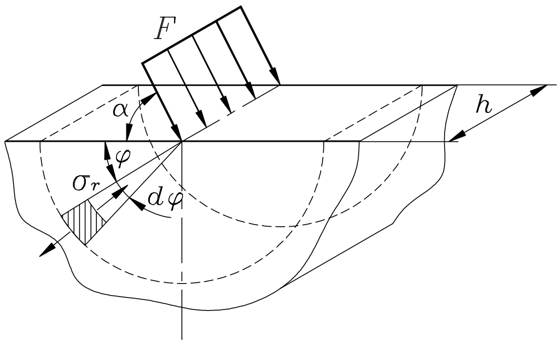


Fig. 2. Application of the contact force *F*

The theoretical basis and complete procedure of force determination by the optical method of caustics is developed and simulated for typical cases of loading conditions. The incident light beam characterised by vector $\vec{r}$ passes through the specimen and projects to the screen into the point characterised by vector $\vec{r}\,'$:

$$\vec{r}\,' = m\vec{r} + z_0 grad\Delta s , \quad (2)$$

where *m* is a scale factor ($m = 1$ for parallel light beams, $m < 1$ for convergent and $m > 1$ for divergent light), z_0 is a characteristic distance and Δs is a difference in the optical path between the unloaded and loaded specimen. The light beam retardation is connected to the principal stresses σ_1

and σ_2, optical constant of caustics c and coefficient of optical anisotropy ε_a ($\varepsilon_a \neq 0$ for optically anisotropic materials), so that the equation (6) leads to

$$\vec{r}' = m\vec{r} + cz_0 h grad\left[\sigma_1 + \sigma_2 \pm \varepsilon_a \cdot (\sigma_1 - \sigma_2)\right]. \quad (3)$$

In general case of transparent light arrangement, double optical effect will appear as outer (index o) and inner (index i) caustics, due to optical anisotropy of material:

$$\begin{aligned} x'_{o,i} &= mr\cos\varphi + \left|cz_0\right| h\left(1 \pm \varepsilon_a\right)\left(\frac{2F}{\pi r^2}\right)\cos(2\varphi + \alpha), \\ y'_{o,i} &= mr\sin\varphi + \left|cz_0\right| h\left(1 \pm \varepsilon_a\right)\left(\frac{2F}{\pi r^2}\right)\sin(2\varphi + \alpha), \end{aligned} \quad (4)$$

valid in the range $0 \le \varphi \le \pi$.

Every point of homogenous light field around a dark shadow on the screen are shown in Fig. 3 as the simulation of the single (outer or inner) optical effect.

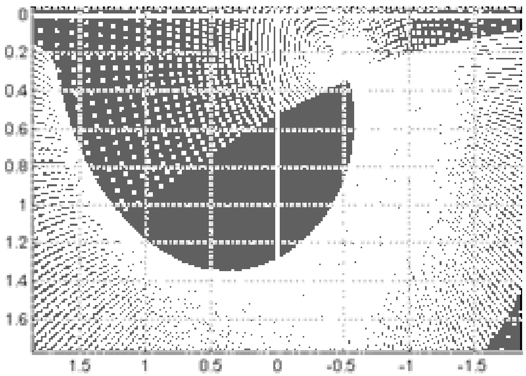


Fig. 3. Simulated contact caustics

The number of the points that surround the dark spot illustrates the light intensity, observing that their density is maximal near the border of the dark spot. Therefore, the singularity of the dark shadow is mathematically described by the Jacobian equation

$$J = \frac{\partial x'}{\partial r} \cdot \frac{\partial y'}{\partial \varphi} - \frac{\partial x'}{\partial \varphi} \cdot \frac{\partial y'}{\partial r} = 0, \quad (5)$$

leading to a parameter r_0:

$$(r_0)_{o,i} = \sqrt[3]{\frac{4F\left|c z_0\right| h\left(1 \pm \varepsilon_a\right)}{m\pi}}. \quad (6)$$

The solution is independent upon the angle φ and represents initial curve on the specimen surface, e.g. the place from where the light beams are projected directly to the caustic curve. A radius r_0 is dependent upon the force F and suitable as a measure of load intensity.

3. EXPERIMENTATION

Experimental equipment consisted of a high energy point light source, a system of lenses, a b/w CCD camera with AD converter and a computer, set up for transparent optical arrangement (Fig. 4).

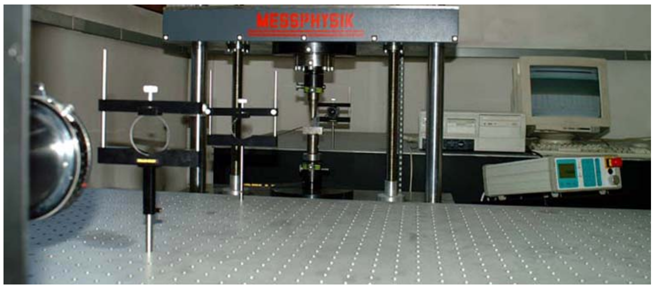


Fig. 4. Experimental setup

Experimental optical effects (Fig. 5) were digitized and automatically analyzed by previously preprogrammed system filters for effective dark spot extraction and quantification. Experiments were performed under the static loading conditions by increasing the load intensity for different types of photoelastic material araldite B of thickness ranging between 3mm and 6mm.

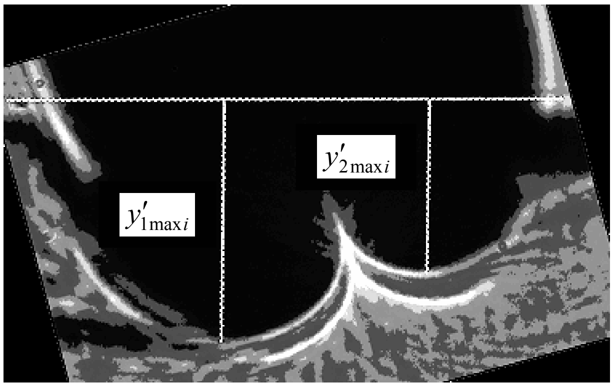


Fig. 5. Experimental contact caustics

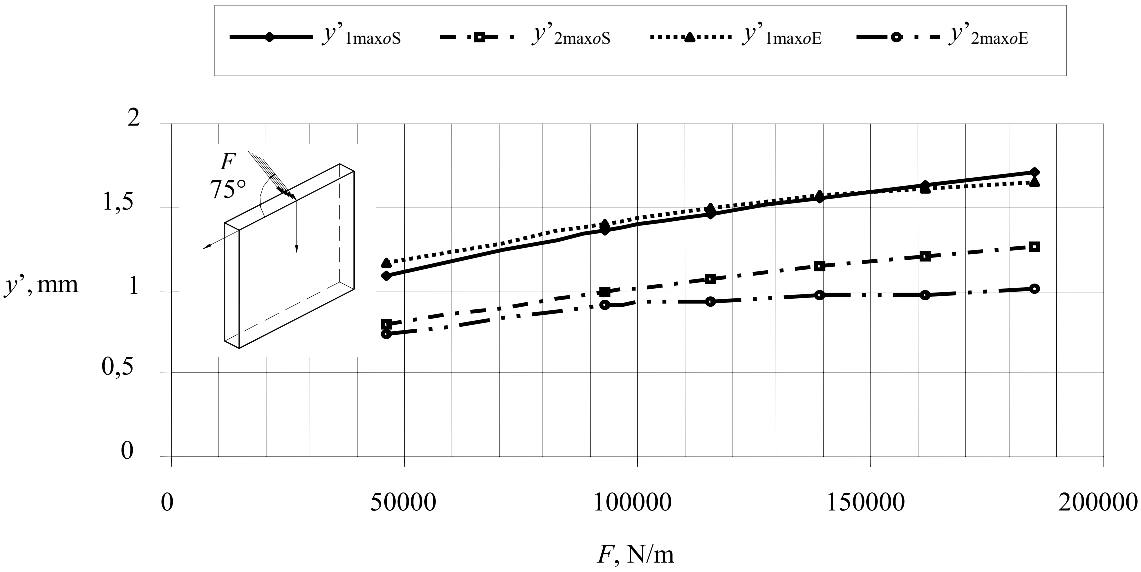
The intensity of the force F which is calculated from the measured value $y'_{1\max}$:

$$F = \left[\frac{y_{1\max o,i}}{\sin\varphi_1 + \frac{1}{2}\sin(2\varphi_1 + \alpha)}\right]^3 \left[\frac{\pi}{4m^2\left|c z_0\right| h\left(1 \pm \varepsilon_a\right)}\right]. \quad (7)$$

The angle α is calculated from simulated caustic curves. The ratio of the extremes of experimental coordinates $y'_{1\max}$ and $y'_{2\max}$ and their simulated values (obtained by specific parameters φ_1 and φ_2) is the same for unique value of α:

$$\frac{y'_{1\max}}{y'_{2\max}} = \frac{\sin\varphi_1 + 0.5\sin(2\varphi_1 + \alpha)}{\sin\varphi_2 + 0.5\sin(2\varphi_2 + \alpha)}. \quad (8)$$

The measuring procedure was established when the experimental optical effects taken and processed in laboratory conditions were compared to the simulated ones [3]. Diagram of different force inclination for different types of body contact is given in Fig 6.



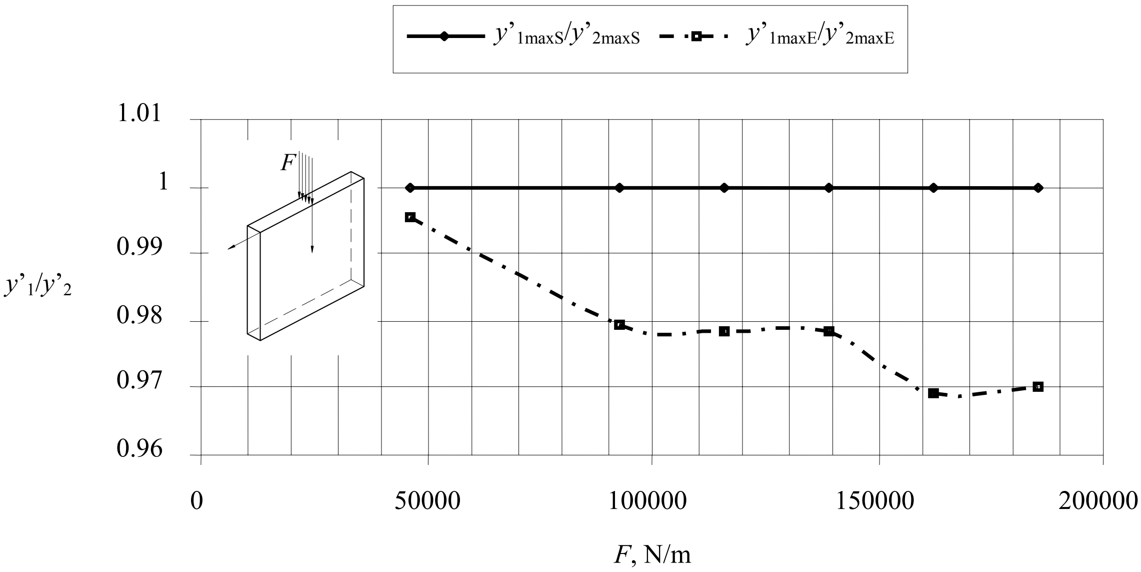


Figure 6. Comparison between experimental (E) and simulated (S) caustics coordinates

The analysis of diagrams shown in Fig 6.:

1. Experimentally measured maximum outer coordinates ($y'_{1\max o\mathrm{E}}$, $y'_{2\max o\mathrm{E}}$) are compared to the simulated values ($y'_{1\max o\mathrm{S}}$, $y'_{2\max o\mathrm{S}}$) for the contact force inclined by 75° (5π/12). The matching with the simulated values is very good for the diameter $y'_{1\max o\mathrm{E}}$ that is ussualy used in (7) for the force determination.

2. A comparison of ratios of experimentally measured and simulated characteristic coordinates for the contact force perpendicular to the surface. The experimental values are very similar to the ideal case of contact when coordinates $y'_{1\max\mathrm{E}}$ and $y'_{2\max\mathrm{E}}$ are the same for such case of loading.

Experimentally determined forces show the maximum difference of 3% to the simultaneous simulations.

The presented analysis of the contact between two elastic bodies introduces the procedure of force determination by the method of caustics. As in contact problems the amount of the applied force as well as its inclination (e.g. in the case of friction normal and tangential component of the force) can be identified and measured from the optical effect (dark spot) on the screen.

4. ADVANCED APPLICATION TO COMPOSITES

Objectives of the research in future are the development of more sophisticated procedures of contact force determination that will enable the extension of the method of caustics to mechanically anisotropic materials represented by composites that are of orthotropic structure.

The presumptions of such an invented method and the expected results are:

1. The equations for the stress-strain state in the region near the point of force application region in anisotropic body should be taken into account.
2. The elastic constants in composites for the usual orthotropic mechanical structure have significant mismatch in the directions of the principal axes of orthotropy *L* and *T*.
3. Shape of singular caustic curves will be changing for different angles of force inclination; size of caustics is proportional to the force intensity.
4. Proper evaluation of the experimental optical effects is based on the properly simulated unit caustics for each position of the principal axes of orthotropy.

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