Signal de-noising using wavelet transform and other methods

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Abstract Using wavelet transform (WT) for increasing signal-to-noise ratio (*SNR*) of discrete-time signals corrupted by additive noise is explained and compared with some other techniques (averaging, frequency filtration, correlation). Signal processing for de-noising is applied to basic periodical signals and repeated transients (in non-destructive ultrasonic testing of welds, where presence of flaws should be detected). Results of both computer simulations and measurements are reported, and some best suitable wavelets, levels of signal decomposition and methods and parameters of thresholding are given. A new efficient method of wavelet thresholding suitable for ultrasonic flaws detection in welds testing is described as a part of practical wavelets *SNR* enhancement (*SNRE)* application, and correlation function used for the same purpose is also described.. Wavelet Toolbox of MATLAB environment is used both for computer simulations and practical signal de-noising.

I. Introduction

Measured signals are in practice mixtures of useful signal and noise. Noise is most frequently additive, and could be thermal noise, noise of electronic structures, quantization noise or external noise connected to the measuring circuits by electromagnetic induction or by capacitive coupling. Noise is usually stochastical signal, which could be described by its probability distribution and (auto)correlation function or power (or energy) spectral density. If the measured signal is discrete in time, then correlation and covariance functions are two-sided time sequences and power spectrum density is periodic time-discrete sequence.

Relation between levels of signal and noise is described by a well-known parameter signal-to-noise ratio (*SNR*) expressed usually in dB and found from ratios of signal and noise powers or RMS values. Finding *SNR* values and methods for increasing the *SNR* by signal processing is an important task in measurement and signal processing, and various methods of it were proposed (e.g. [1- 4]). To find the *SNR*, powers or RMS values of signal and noise has to be found. Using a method for lowering noise means increasing the *SNR* value.

The aim of this contribution is to evaluate the efficiency of wavelets as modern mathematical tool usable for suppressing additive noise mixed with one-dimensional signal and so increasing the signal-to-noise ratio (*SNR*). Wavelet Toolbox for use with MATLAB [5] and other MATLAB environment products were used both for simulations and measurements. Basic wavelets theory is explained in many books, e.g. in [6, 7] and it will not be repeated here.

De-noising algorithms based on wavelets, signal averaging and filtration using frequency-selective digital filters is applied on basic periodical signals and the efficiency of individual methods of *SNR* enhancement (1) is compared for selected wavelets and filters.

Effective de-noising of *sinusoidal signal* can be achieved also by correlation filtration (by computing autocorrelation function of the signal with additive noise, which after delay surpassing the maximum interval of correlation is clean cosinusoid with magnitude $U^2_{\max}/2$, where $U_{\max}$ is the maximum value of the sinusoidal signal, and with frequency identical with frequency of signal). Unfortunately, information about signal phase is lost by using this method. Correlation filtration using cross-correlation function was successfully used in ultrasonic material testing and is compared with wavelet transform de-noising in the same application (ultrasonic flaws detection in welds testing). That is a practical example of de-noising a repeated transient signal. A new and efficient method of threshold setting for this case is briefly described and numerical values of signal-to-noise enhancement (*SNRE*, see (1)) are given for selected wavelets.

II. Basic periodical signals de-noising

We have verified the efficiency of various methods in periodical signals de-noising on low frequency (1 kHz) sinusoids, rectangular and triangle periodical waveforms mixed with additive noise so that a prescribed *SNR* is achieved. These waveforms were inputs to various de-noising algorithms and values of *SNRE* (*SNR* enhancement) were compared. The *SNRE* is defined as

$$SNRE = SNR2 - SNR1 \quad \text{(dB)} \tag{1}$$

where *SNR2* is *SNR* of output signal and *SNR1* is *SNR* of input signal. Principle of this signal de-noising is shown in Fig.1, where H(z) is Z-transform transfer function of the de-noising LTID system or numerical algorithm and h(n) is its impulse response.

$$x_1(n) = s_1(n) + n_1(n) \longrightarrow \boxed{H(z),\ h(n)} \longrightarrow x_2(n) = s_2(n) + n_2(n)$$

Fig.1 Increasing *SNR* by applying signal to an LTID system or numerical algorithm.

The *wavelet transform de-noising* basic procedure consists of three steps: *multiple-level signal decomposition* on approximation and detail coefficients for chosen level of decomposition N (performed using the DWT – Discrete Wavelet Transform), *thresholding of detail coefficients* (by selecting threshold for each level from 1 to N and applying soft thersholding), and *signal reconstruction* using the original approximation coefficients of level N and the modified detail coefficients of levels 1 to N (performed using the IDWT – Inverse Discrete Wavelet Transform). The choice of the wavelet type, level of decomposition, and of the type of thresholding (soft or hard) and way of finding threshold levels influence the efficiency of *SNRE* for the given signal.

Apart from using wavelets also frequency filtration by means of low-pass (LP) and band-pass (BP) filters, and signal averaging (denoted also as summation averaging) covering selected number of signal periods was evaluated. *Filtration using frequency selective filters* is based frequency domain signal processing, namely on suppressing noise and (as much as possible) not influencing signal. It can be very efficient for sinusoidal signal if band pass filter with very narrow pass band placed at the signal frequency is used or LP filter is applied on low frequency sinusoid. This method is not very suitable for other periodic signals, since the used filter suppresses many higher harmonic components and changes so both waveform and power of the signal. Nevertheless sometimes corner frequency of LP filter can be chosen so that acceptable change in signal occurs and comparatively high *SNRE* is achieved (optimum between *SNRE* and signal deformation should be found). We have checked both FIR (M=12 and M=20) and IIR (Chebyshev1, M=3) LP and BP filters. The highest SNRE values together with corresponding filter type are given in Tab. 1. sinusoidal signal and Tab. 2 for rectangular pulse train. *Summation averaging de-noising* consists of coherent sampling of integer number M of periods of signal (corrupted with noise), and calculation of arithmetic mean of M individual samples each in one period and with identical position in signal period. Each sample of corrupted signal is a sum of signal sample and noise sample. Average of signal samples in the same position in signal period is equal to (ideal) signal sample, and standard deviation of average of corresponding noise samples σ_M is equal to $\sigma_M = \sigma / \sqrt{M}$, where σ is standard deviation of the noise and *M* is number of coherently sampled signal periods.

Values of variance of noise (white and normally distributed) added to input signal were chosen according to the defined *SNR1* (20 dB for reported results). Output signal and noise powers (and consequently values of *SNR*) were estimated using auto-correlation functions values in origin ($R_{XX}(0) = P_S + P_N$) and in local maximums ($R_{XX}(k.T_{SIG}) = P_S$, T_{SIG} being signal period). Since the values of several first $R_{XX}(k.T_{SIG})$ were found not to be constant (due to additive noise influence), averaging of five local maximums was used. A part of preliminary simulation results was presented in [8].

The algorithms used for *SNR* enhancement should in ideal case not influence the signal $s_1(n)$. If there is for powers of signals in Fig.1 $P_{s1} = P_{s2}$, then

$$SNRE = 10\log\frac{P_{s2}}{P_{n2}} - 10\log\frac{P_{s1}}{P_{n1}} = 10\log\frac{P_{n2}}{P_{n1}} \tag{2}$$

In practice there is often $P_{s1} \neq P_{s2}$, especially if nonsinusoidal periodic signals are de-noised and frequency selective filters are used, since frequency spectrum of signal and noise are overlapping. Influencing the signal by the used algorithm can be found using clean signal ($x_1(n)=s_1(n)$) at its input and finding the error signal $s_e(n)=x_2(n)-x_2(n)$.

Many wavelet types, decomposition levels and thresholding parameters, different LP and BP filters and averaging of different numbers of periods of signal sampled were examined in de-noising. Results corresponding to the highest achieved values of *SNRE* are presented in Tab. 1. Wavelet abbreviations correspond to notation used in Wavelet Toolbox for use with MATLAB [5] ('bior' is biortogonal wavelet, 'dmey' is discrete approximation of Meyer wavelet, 'sym' is symlet wavelet, 'db' id Daubechies wavelet, 'coif' is wavelet coiflet, 'Haar' is haar wavelet). Soft thresholding allowed higher *SNRE* than hard thersholding and was therefore applied in all cases.

Tab.1 *Sinusoidal signal* de-noising - *SNRE* comparison (*SNR1* = 20 dB)

Wavelet Transform					
wavelet	bior1.5	dmey	sym7	db7	coif4
level of decomposition	5	5	5	5	5
method of thesholding	heursure sln	heursure sln	heursure sln	heursure sln	heursure sln
SNRE (dB)	20.5	17.3	15.0	14.9	14.8

Filtration			
filter type	FIR LP(M=20,fc=1.2kHz)	IIR LP(Cheby1,M=3)	IIR BP(Cheby1, N=4, 0.8÷1.2kHz))
SNRE (dB)	11.9	12.2	9.4

Signal averaging				
Number of periods	4	8	16	32
SNRE (dB)	11.4	13.5	18.3	21.9

Tab.2 *Rectangular signal* de-noising - *SNRE* comparison (*SNR1* = 20 dB)

Wavelet transform					
wavelet	db4	bior3.5	bior3.9	Haar (i.e. db1)	bior 3.5
level of decomposition	5	5	5	5	2
method of thesholding	heursure sln	heursure sln	heursure sln	rigrsure sln	heursure mln
SNRE (dB)	3.33	3.2	2.9	2.6	2.6

Filtration (filters with upper corner frequency 10kHz were used to preserve roughly signal wave shape)			
filter type	FIR LP (M=12)	IIR LP (Cheby1,M=7)	FIR BP (N=360)
SNRE (dB)	5.9	4.8	6.8

Signal averaging				
Number of periods	4	8	16	32
SNRE (dB)	2.7	9.7	15.0	22.2

III. Wavelet transform and correlation –based signal de-noising in ultrasonic material testing

Signal de-noising for flaws detection in ultrasonic non-destructive testing of welds is an example of practical use of WT and cross correlation. Unlike in paragraph II, signal here is of transient nature, and a suitable mother wavelet or another mathematical function (if using correlation filtration) with the shape similar to that of the ultrasound signal can be selected for analysis. Evaluation of flaws is based on decision if a reflector (cause of reflected wave) is a flaw and noise reduction is often a condition of this evaluation**.** Noise is caused mainly by scattering of the signal at the inhomogenities in the structure of the material and by the electronics.

A special gauge was made for experiments on *SNRE*, made of two welded metal sheets 9.2 mm in thickness. Gauge material was the one used for airplane engines construction and it is very granular hard alloy. The flaws artificially manufactured on the special gauge were seven holes made by spark technology with diameter of 0.5 mm (inner five holes) and 0.7 mm (the two outer holes). Both metal blocks were welded together using electron beam.

Traditional techniques for reducing ultrasonic signal noise are based on selecting the optimum frequency of the acoustic wave, on ultrasonic probe construction and on using low-noise electronic circuits.

In this *WT de-noising* application, wavelets „db" (Daubechies), „bior" (biortogonal) and „sym" (symlet) were used, since they are similar to the wave shape of the flaw-generated real ultrasonic signal. Threshold method "sqtwolog" (see [5]) was used. Achieved values of *SNRE* are given in table 3 for the wavelets mentioned in the first

Tab.3 *SNRE* for selected wavelets and thresholding using standard algorithms

	thresholding method (see [5])		
wavelet	rigrsure	heursure	sqtwolog
bior 3.9	19,0	22,5	29,8
sym3	17,6	22,1	27,1
sym8	17,2	20,4	24,8

column and corresponding to the highest values of the *SNRE*.

The efficiency of the *WT method of de-noising* as compared to basic procedure can be considerably increased by repeated computation (using combinatorial analysis) of optimum threshold value for wavelet decomposition of every detail coefficient up to the decomposition level 5. The combination of thresholds for all used decomposition levels ensuring highest *SNRE* was be found numerically by computer simulation using signal corresponding to real signal, and afterwards these threshold values were used for de-noising of real ultrasonic signals [9].

The results of the measurements for wavelet sym3 are given in Fig.2.

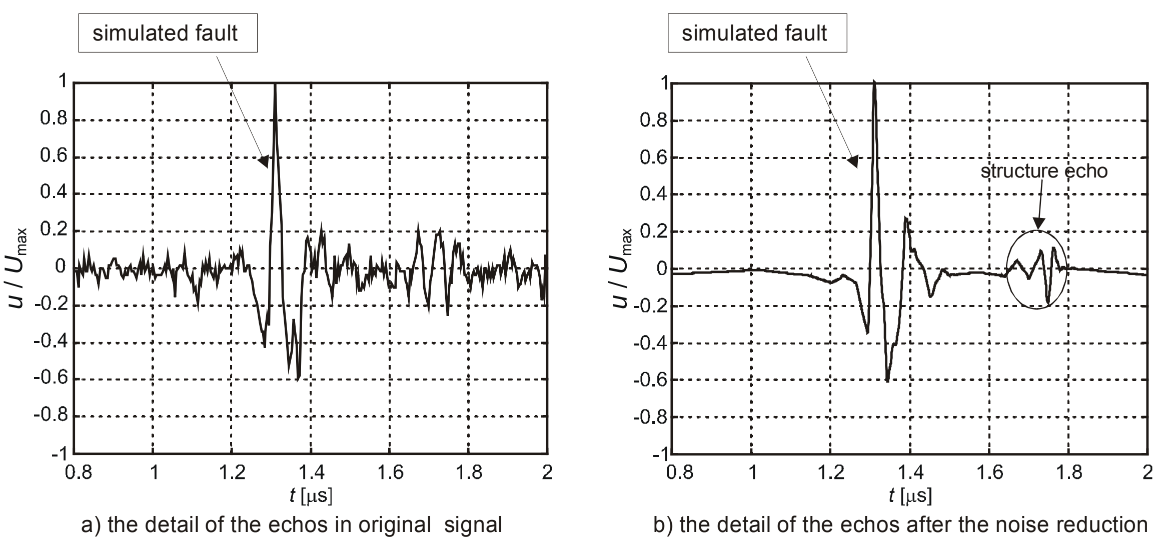

Fig. 2: Detail of ultrasonic signal on special gauge and signal de-noising (*SNRE* ≈ 54 dB).

Correlation filtration was also tested for signal de-noising in ultrasonic material testing. Cross correlation function of measured and simulated signal of the ultrasonic impulse was computed according to (4)

$$\hat{R}_{x,y} = \frac{1}{N} \sum_{n=1}^{N} x(nT_s) \cdot y(nT_s + rT_s), \tag{4}$$

where T_s is sampling period,
$x(nT_s)$ is the ultrasonic measured signal,
$y(nT_s)$ is the simulated signal of the ultrasonic impulse,
N is number of samples,
and there is $y(nT_v) = 0$ for $nT_s < 0$, $r = -N, \ldots, N$.

Experiments were performed on the same special gauge. The simulated signal (representing reflected signal) was modeled as Gaussian pulse modulated by cosine function according to (5)

$$y(nT_v) = \cos(2\pi f nT_v - \varphi_0) e^{-\pi(\sigma nT_v - \varphi_0)^2} \tag{5}$$

where optimal choice phase shift φ_0 was $7\pi/2$ and frequency f and damping coefficient σ were found as coordinates of maximum's in the 3-D figure of $SNRE(f, \sigma)$ and were found to be f=20 MHz (which corresponds to the 20 MHz ultrasound probe used) and σ = 20.

The algorithms was tested also on real data using the special gauge mentioned above, and a detail of the result shoving efficiency of de-noising based on correlation filtration is shown in Fig.3.

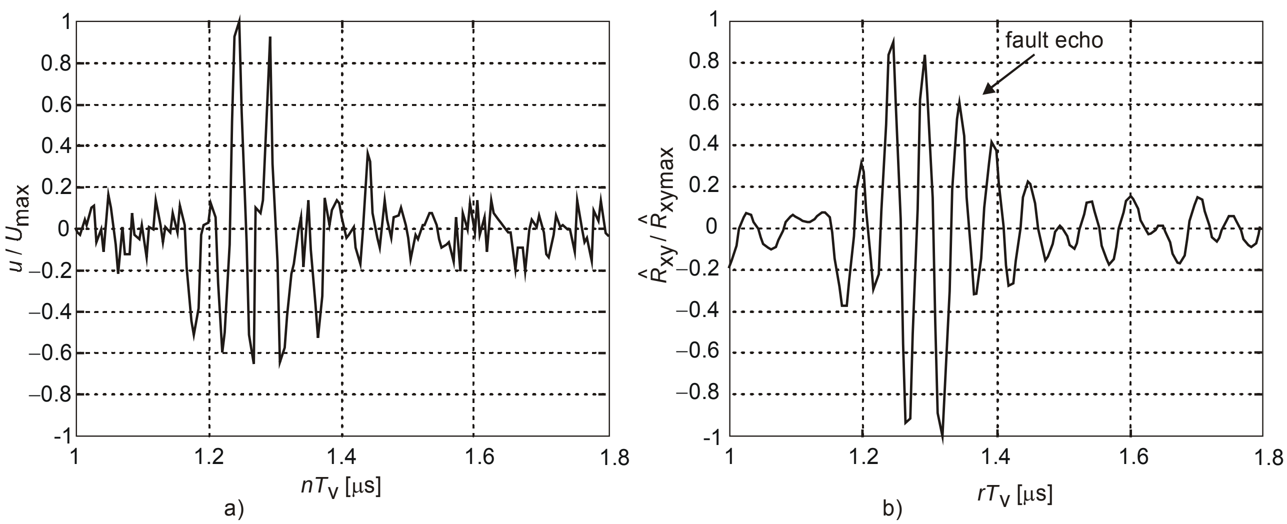

Fig. 3 Detail of ultrasonic signal on special gauge and signal de-noising (a – raw signal, b – correlation filtered signal)

IV. Conclusions

Wavelet transform is usable for *SNR* enhancement for both basic periodical signals and in special applications as ultrasonic material testing. Necessity of experimental seeking wavelet type, decomposition level and thresholding procedure for optimum performance for given type of signals is its drawback. Its advantage compared to frequency filtration de-noising is possibility to use this method effectively also in case of overlapping frequency spectra of signal and noise. Applying suitable decomposition and thresholding parameters, rising times of signal jumps can be preserved here. Possible deformation of signal depends on thresholding level and signal frequency spectrum – if it contains high frequency components, thresholding of detail coefficients has to be carefully selected, since signal high frequency components could be also filtered out. *SNRE* values gained from WT de-noising depend also on *SNR1* value (*SNR* of the original signal corrupted with noise).

Efficiency of filtration of basic periodical signals corrupted with noise using *frequency selective filters* is limited by signal deformation caused by the used filter. This deformation depends on filter corner frequency (or frequencies), power spectrum density of the additive noise, and frequency spectrum of the signal, and leads in general to signal deformation, typically (by using LP or BP filter) to slowing jumps in signal. The *SNRE* values given in Tab. 2 should be therefore taken with care, since even if they are higher than those gained by WT de-noising of rectangular signal, signal deformation is usually lower when using wavelets. This method is suitable especially for sinusoidal signals.

The *SNRE* value increases with increasing number of sampled periods when *signal averaging* coherently sampled periodical signal corrupted by noise. In our experiments this proved to be the most efficient method (for 32 signal periods averaged).In practical applications the efficiency of this method is limited by possible instability of signal period (our signals were either simulated or generated by signal generator based on frequency synthesis) and by possible difficulties in signal/sampling synchronization for signals with low SNR values.

In *ultrasonic material testing* by WT application, using combinatorial analysis for selection of threshold values in individual levels of signal decomposition allows to increase considerably the attainable value of SNR enhancement. Finding parameters of mathematical model of reflected ultrasonic signal for maximum *SNRE* before actual signal de-noising (by analyzing the 3D figure of SNRE as function of those parameters) and computing the cross-correlation function of the measured ultrasonic signal with the optimum simulated signal of the ultrasonic pulse allows very effective signal de-noising. The scattering at the inhomogeneities in the structure of the material causes strong coherent noise and the signal de-noising correlation method increases frequency component 20 MHz.

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