

# USING LOWER CLASS WEIGHTS TO CORRECT AND CHECK THE NONLINEARITY OF BALANCES

Timothy Changlin Wang<sup>1</sup>, Qihao Yuan<sup>2</sup>, Arthur Reichmuth<sup>2</sup>

<sup>1</sup>Mettler-Toledo Instruments (Shanghai) Co. Ltd, Shanghai, P R China

<sup>2</sup>Mettler-Toledo GmbH, Greifensee, Switzerland

## ABSTRACT

Any electronic balance has more or less nonlinearity. This paper presents a method to correct and check the nonlinearity by using two sets of lower class weights. Some of the key causes of the nonlinear error are described first. Common-meet parabola nonlinear pattern were studied in detail. A correcting procedure, using a set of lower class or even no class weights is then described. The correcting coefficient is given in the paper in an apparent form. Simulation and test results showed the nonlinear error reduced dramatically with the method. A check process is required before a balance goes to the market to test whether the final nonlinearity is within the tolerance. A check procedure, namely differential method, is presented. The method is accurate enough by using a second set of lower class weights. Low cost, high efficiency and good quality can be achieved in balance production by using the method presented.

## 1. INTRODUCTION

The linearity describes how well a balance is able to follow the linear relation between the load and the display value of the balance. The ideal characteristic weighing curve is a straight line between the zero and maximum loads. The nonlinearity defines the maximum positive or negative deviation of the displayed value from that of the ideal characteristic of a straight line. Every electronic balance shows more or less nonlinearity. The key causes of nonlinearity are described below.

### 1.1 Mechanical deformation

The section area of the elastic beam in a strain gauge type balance changes when the load changes. This will cause the nonlinear relationship between the load and the strain produced. The level beam in an MFR (magnetic force restoration) type of balances will deform when load added. Then the ratio of the beam will change. This will also cause nonlinearity.

### 1.2 Electronic

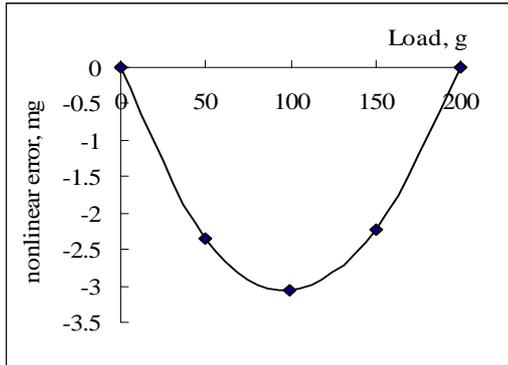
The output of the bridge in a strain gauge load cell is theoretically not linear with the strain felt. In case the bridge is not symmetric, or the load is too large, the nonlinear error will increase. The bad common mode rejection of a comparator may cause nonlinearity in a PWM related A/D converter, as the rejection ratio is related, but not linearly related to the common mode input, which is proportional to the load usually. Working point related turn on/turn off time delay of a transistor will influence the PWM results. There is a phase delay if the end point impedance of the signal transmitting line between the load cell and the A/D converter does not match with the line impedance. This phase delay will vary, but not linearly vary with load changes, and will cause nonlinearity.

### 1.3 Magnetic

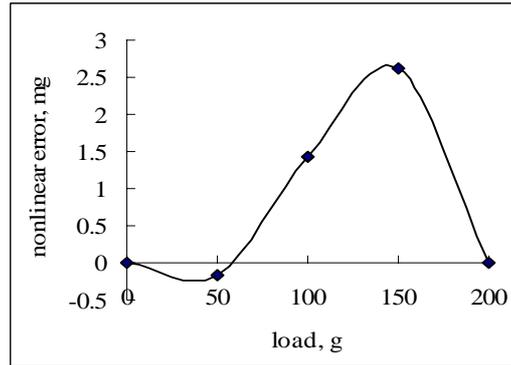
The compensation force is supposed to be linearly related with the current in the coil, which is located in a magnetic field in an MFR type of balances. However, as the current will influence the magnetic field, their reactions will cause more or less nonlinearity in a balance.

In practice, the nonlinear characteristic (or the nonlinear pattern) of a correctly designed and properly manufactured balance should be relatively stable. It should have no discontinuities in the working range and the pattern should be as simple as a parabola before correction [1].

Figure 1 shows the result of a correctly designed and properly manufactured balance before nonlinearity correction. With a modern electronic balance, this nonlinear error is usually corrected by the software before the value is displayed. It will improve the performance dramatically to use parabola correction, or 3-point linearization, as described in this paper. Figure 2 however, shows the nonlinear characteristic of a badly designed balance model before correction. With this pattern, 3-point correction will not work properly.

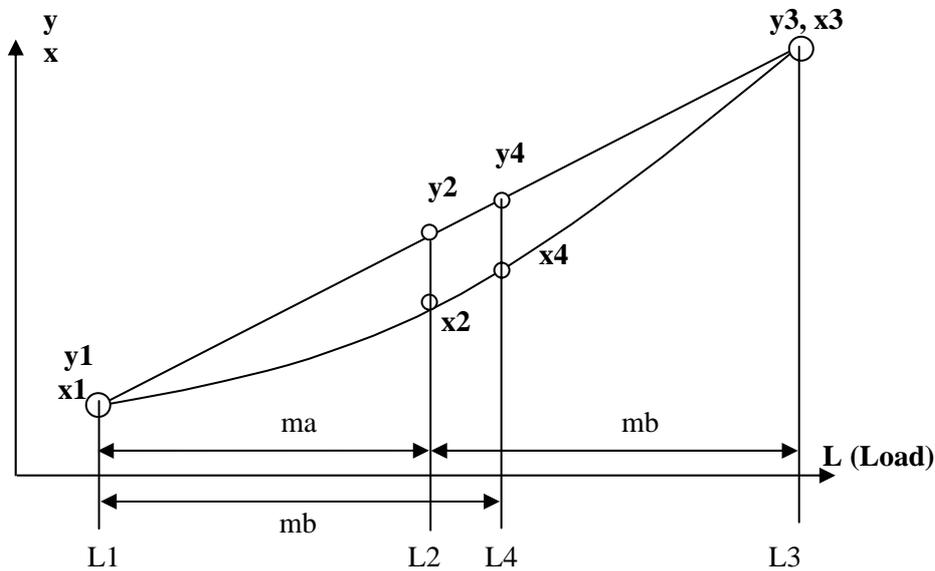


**Figure 1:** Correctly designed balance (before correction)



**Figure 2:** Badly-designed balance model (before correction)

## 2. LINEARISATION



**Figure 3:** 3-points linearization

Procedure

Zero Load:  $L1=L1$ , A/D out:  $x1$   
 1/2 Load, first:  $L2=L1+ma$ , A/D out:  $x2$   
 Full load:  $L3=L1+ma+mb$ , A/D out:  $x3$   
 1/2 Load, second:  $L4=L1+mb$ , A/D out:  $x4$

We define the relative load as:

$$r = \frac{L - L_1}{L_3 - L_1} \quad (1)$$

$$r_a = \frac{L_2 - L_1}{L_3 - L_1} = \frac{m_a}{m_a + m_b} \quad (2)$$

$$r_b = \frac{L_4 - L_1}{L_3 - L_1} = \frac{m_b}{m_a + m_b} \quad (3)$$

$$r_a + r_b = 1 \quad (4)$$

Assuming the parabola pattern of nonlinearity,

$$x = r(x_3 - x_1) + x_1 + NL[1 - (\frac{r-0.5}{0.5})^2] \quad (5)$$

Where, NL is the initial nonlinear error in the half load between L1 and L3. Let

$$t = \frac{x - x_1}{x_3 - x_1} \quad (6)$$

$$t = r + \frac{NL}{x_3 - x_1} [1 - (\frac{r-0.5}{0.5})^2] \quad (7)$$

If  $t_a = \frac{x_2 - x_1}{x_3 - x_1}$ ,  $t_b = \frac{x_4 - x_1}{x_3 - x_1}$ , then

$$t_a = r_a + \frac{NL}{x_3 - x_1} [1 - (\frac{r_a - 0.5}{0.5})^2] \quad (8)$$

$$t_b = r_b + \frac{NL}{x_3 - x_1} [1 - (\frac{r_b - 0.5}{0.5})^2] \quad (9)$$

As  $r_b = 1 - r_a$ , one can find,

$$r_a = \frac{1 + t_a - t_b}{2} = \frac{1}{2} + \frac{1}{2} \frac{x_2 - x_4}{x_3 - x_1} \quad (10)$$

Let the correcting formula be a parabola also:

$$y = t(x_3 - x_1) + x_1 + A[1 - (\frac{t-0.5}{0.5})^2] = x + A[1 - (\frac{t-0.5}{0.5})^2] \quad (11)$$

Let,

$$u = \frac{y - x_1}{x_3 - x_1} \quad (12)$$

Then,

$$u = t + \frac{A}{x_3 - x_1} [1 - (\frac{t-0.5}{0.5})^2] \quad (13)$$

If

$$u_a = \frac{y_2 - x_1}{x_3 - x_1} \quad (14)$$

And let  $u_a = r_a$ , to determine A=Aa

$$A_a = \frac{(r_a - t_a)(x_3 - x_1)}{1 - (\frac{t_a - 0.5}{0.5})^2} = \frac{r_a(x_3 - x_1) - (x_2 - x_1)}{1 - (\frac{t_a - 0.5}{0.5})^2} \quad (15)$$

$$y = x + A_a \left[ 1 - \left( \frac{t - 0.5}{0.5} \right)^2 \right] \quad (16)$$

Similarly, we assume

$$u_b = \frac{y_4 - x_1}{x_3 - x_1} \quad (17)$$

And let  $u_b = r_b$ , to determine  $A=Ab$ ,

$$A_b = \frac{r_b(x_3 - x_1) - (x_4 - x_1)}{1 - \left( \frac{t_b - 0.5}{0.5} \right)^2} \quad (18)$$

$$y = x + A_b \left[ 1 - \left( \frac{t - 0.5}{0.5} \right)^2 \right] \quad (19)$$

To make the solution symmetry, let

$$A = (A_a + A_b) / 2 \quad (20)$$

$$y = x + A \left[ 1 - \left( \frac{t - 0.5}{0.5} \right)^2 \right], \text{ or}$$

$$y = c_1 + c_2 x + c_3 x^2 \quad (21)$$

Where, the correcting coefficient,

$$c_1 = -x_1 x_3 B \quad (22)$$

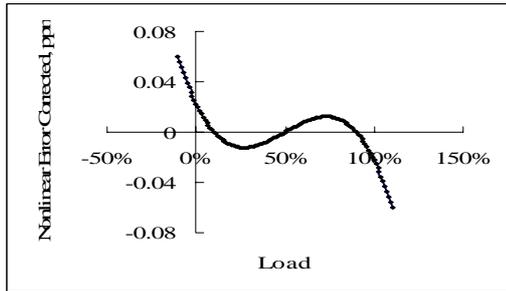
$$c_2 = 1 + (x_1 + x_3) B \quad (23)$$

$$c_3 = -B, \quad (24)$$

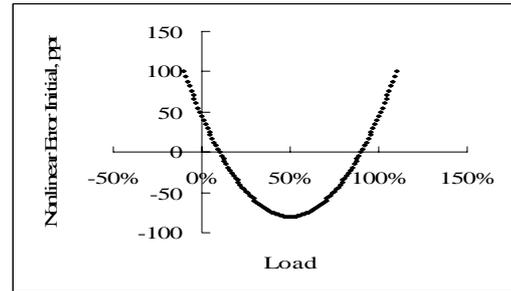
And,

$$B = \frac{4A}{(x_3 - x_1)^2} \quad (25)$$

One can find the correcting coefficient can be obtained in an apparent form. The procedure is very accurate if the nonlinear pattern is parabola. An initial nonlinear error of 100 ppm (from -10% to 110% full load) for example in Figure 4, even if  $m_a$  and  $m_b$  are not accurate at all, such as  $r_a=0.3$ ,  $r_b=0.7$ ,  $L1=10\%$  full load,  $L3-L1=80\%$  full load, the maximum final nonlinear error after correction will theoretically be less than 0.06 ppm (from -10% to 110% full load), as Figure 5 shows.



**Figure 4:** Initial nonlinear error, example



**Figure 5:** Nonlinear error after correction

In case the targeted nonlinear error after correction is 1 ppm, calculation results show that the initial nonlinear error should not be larger than 400 ppm. The procedure is excellent for using lower class, or even no class weights to carry out the linearization. After the correction, one can find, there is no error when the load is  $L1$ ,  $(L1+L3)/2$ , and  $L3$ . So the method is called 3-points linearization. It is easy to estimate the initial nonlinear error with below formula, if  $r_a$  is close to 0.5, and  $L3-L1$  is close to the full load.

$$NL \approx \frac{\frac{1}{2}(x_2 + x_4) - \frac{1}{2}(x_3 + x_1)}{x_3 - x_1} \quad (26)$$

Formula (26) can also be used by the end user to estimate the performance of a balance. Practically, to get the best result, the initial nonlinear error should not be larger than 100 ppm if the requirement of the final error is 1 ppm. L3-L1 should be as large as possible (the full load).  $r_a$  should not be less than 0.3, no greater than 0.7 (the best is 0.5), L1 close to the zero load is the best.

### 3. NONLINEARITY CHECK

After the correction, a check procedure must be followed in balance production to see whether the pattern assumed to be accurate enough, or to check whether the nonlinear error in the whole weighing range is within the specification after the correction mentioned. After 3-points linearization, there are at least three load points for the balance without nonlinear error as mentioned above. It is reasonable to assume that the nonlinear pattern after such linearization is a cubical parabola passing through the named 3 points.

$$NLr = NLM * 20.785 * r * (r - 0.5) * r * (r - 1) \quad (27)$$

Where NLM in ppm is the maximum nonlinear error (when  $r=0.2113$ , or  $r=0.7887$ ). The curve showed on Figure 6 is the said pattern. The procedure is to design a test, to estimate the maximum nonlinear error, and see whether it is within the specified tolerance, 1 ppm for example. Theoretically, this task can be done by using a set of known weights. But the process is costly and low efficient, or even impossible in some cases. We can design a 5-segment check procedure according to the differential method by using a set of 5 weights ( $W1 \dots W5$ ), each of which is approximately 20% of the full load, and use the first one as the reference weight ( $W_r = W1$ ).

1. 0% load, display Z1
2. 20% load:  $W_r$ , display Z2
3. 20% load:  $W_2$ , display Z3
4. 40% load:  $W_2 + W_r$ , display Z4
5. 40% load:  $W_2 + W_3$ , display Z5
6. 60% load:  $W_2 + W_3 + W_r$ , display Z6
7. 60% load:  $W_2 + W_3 + W_4$ , display Z7
8. 80% load:  $W_2 + W_3 + W_4 + W_r$ , display Z8
9. 80% load:  $W_2 + W_3 + W_4 + W_5$ , display Z9
10. 100% load:  $W_2 + W_3 + W_4 + W_5 + W_r$ , display Z10

Then calculate the difference or the segment increase corresponding to  $W_r$ :

$$D1 = Z2 - Z1 \quad (28)$$

$$D2 = Z4 - Z3 \quad (29)$$

$$D3 = Z6 - Z5 \quad (30)$$

$$D4 = Z8 - Z7 \quad (31)$$

$$D5 = Z10 - Z9 \quad (32)$$

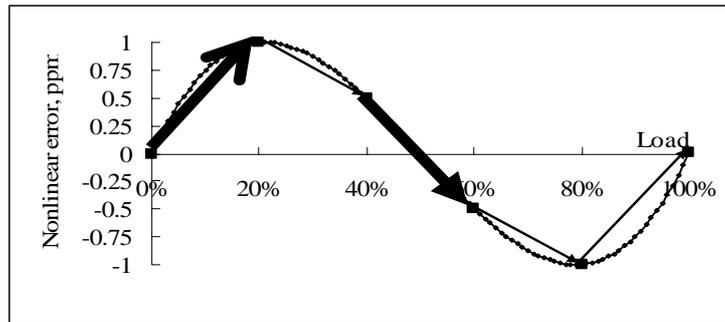
The nonlinear curve can be estimated as

$$NL_i = D_i - \text{average } D_i \quad (33)$$

And maximum nonlinear error,

$$NLm = k * \frac{\max_{i=1}^5 D_i - \min_{i=1}^5 D_i}{2} \quad (34)$$

Where k=1.002 makes theoretically the formula (34) fit. The two boldface arrows in Figure 6 represent the max(Di)-average(Di), and min(Di)-average(Di). Sensitivity study shows, even if Wi is (20% full load) +/- 5%, k=1.033 is safe enough.



**Figure 6:** 5-segment nonlinearity check

One may consider a sine nonlinear curve other than the cubical parabola pattern. That is to assume:

$$NLr = NLm * \sin(2\pi * r) \quad (35)$$

The maximum nonlinearity error occurs when  $r=0.25$  or  $r=0.75$ , then a 4-segments nonlinear check (using 4 weights each 25% full load approximately) may be used (and  $k=1.00$ ). If 5-segments check is adopted any way,  $k=0.94$  will be good enough. Whatever cubical parabola or sine pattern, we need use in all cases  $k=1.033$  for 5-segments check procedure in formula (34). The result is towards more safe direction. The weights need not be accurate, even no class weights are OK. For analytical balances, the weights with a tolerance of 5% are good enough in most cases.

The comparison with other method based on the traditional nonlinearity definition showed, that the product quality of linearity related was guaranteed with the presented method. At the same time the method is low cost, high efficient in the production.

During the design phase however, it is required to do much more test to prove that the nonlinear pattern presumed is accurate enough.

## REFERENCES

- [1] Arthur Reichmuth: Non-Linearity Of Laboratory Balances And Its Impact On Uncertainty. NCSL Workshop and Conference, Toronto, Canada, July 17-20 2000.
- [2] Heinz Rutishauser, Arthur Reichmuth: The New METTLER AT Analytical Balance. Mettler Toledo Company Publication 1988 (ME-720782)

## Address of the Authors:

Timothy Changlin Wang, Mettler-Toledo Instruments (Shanghai) Co. Ltd, 589 Gui Ping Road, Shanghai 200233, P R China. [Timothy.wang@mt.com](mailto:Timothy.wang@mt.com)  
 Qihao Yuan & Arthur Reichmuth, Mettler-Toledo GmbH, Im Langacher, P.O Box Lab Tec CH-8606 Greifensee, Switzerland. [qihao.yuan@mt.com](mailto:qihao.yuan@mt.com) & [arthur.reichmuth@mt.com](mailto:arthur.reichmuth@mt.com)