Calibration of High Frequency Instruments- Evaluation of Uncertainty by Monte Carlo Approach

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Abstract – **The calibration of high frequency instruments, like oscilloscopes or frequency counters, is a metrologically very intensive task, due to: complex measurement procedures, unattested SI measurement traceability chain for high frequencies signals, and numerous uncertainty contributions. These calibrations are to some extent prescribed in the Euramet cg-7 Guideline. The uncertainty models for calibration of oscilloscopes, introduced by the Laboratory for Electrical Measurements at the Ss. Cyril and Methodius University in Skopje are presented. An original software developed in LabVIEW™, MonteCalc Uncertainty Toolkit, based on both methods, the GUM and the stochastic Monte Carlo approach for uncertainty evaluation is applied on experimental data from laboratory calibration of a high frequency oscilloscope with over 500 MHz range. The results from the two methods are compared and discussed. Based on the uncertainty propagation distribution gained, conformity assessment of the artefact of calibration in particular measurement points, is made against prescribed decision making rules embedded in the MonteCalc Uncertainty Toolkit.**

Keywords: **Calibration of high frequency instruments, measurement uncertainty, GUM, Monte Carlo, LabVIEW™.**

I. INTRODUCTION

The calibration of high-frequency instruments is a critical aspect of ensuring accurate measurements in various scientific, engineering and industrial applications, [1]. High-frequency instruments, such as vector network analyzers, microwave measurement systems, oscilloscopes or counters, are essential tools in fields ranging from telecommunications, electronics to materials science, in particular in fields related to Industry 4.0, [2]. However, one major hurdle in high-frequency instrument calibration is establishing a continuous measurement traceability chain, [3]. This is a recognized international metrological issue, evidenced by published data showing limitations in the traceable high-frequency upper limits of National Metrology Institutes' (NMIs) calibration and measurement capabilities (CMCs) within the International Bureau of Weights and Measures' (BIPM) Key Comparison Database (KCDB), [3]. Furthermore, accurately evaluating the uncertainty of calibration results at very high frequencies involves computationally intensive modeling due to numerous significant and often unknown factors, [4]-[8]. Validating newly developed calibration methods is also difficult due to the scarcity of appropriate metrological facilities, reduced options for proficiency testing schemes, inter-laboratory comparisons, and other crucial quality assurance measures that also ensure the confidentiality of calibration results. The Monte Carlo method, as described in the Guide to the Expression of Uncertainty in Measurement (GUM) Supplement 1, [9], [10], uses random sampling techniques to propagate uncertainties through complex measurement models, [11]. This is particularly advantageous for high-frequency instruments, where traditional analytical methods may fall short due to the nonlinear and stochastic nature of the measurements, [12]. By simulating a large number of possible measurement outcomes, the Monte Carlo method provides a robust statistical framework for estimating the uncertainty associated with calibration functions. Recent studies have demonstrated the effectiveness of the Monte Carlo method in various high-frequency measurement scenarios. For instance, in [12] its application in the calibration of microwave systems, shows that it can significantly reduce bias and improve the accuracy of uncertainty estimates. Similarly, in [11] the use of Monte Carlo simulations in evaluating the uncertainty of calibration functions, emphasizes its superiority over traditional methods. Other papers, [13], [14] confirm the adequacy of these approaches in diverse high frequency calibration cases.

In the paper, the measurement uncertainty model for the calibration of oscilloscopes developed by the Laboratory for Electrical Measurements (LEM) at the Ss. Cyril and

Methodius University in Skopje will be presented. An original software MonteCalc Uncertainty Toolkit, [15], developed in LabVIEW™ based on both methods, the mainstream GUM, [9], [16], and the stochastic Monte Carlo approach, [10] for uncertainty evaluation will be applied. The results derived from the two methods will be compared and discussed by using experimental data from laboratory calibration of an oscilloscope of very high frequency range of over 500 MHz, [1]. Based on the uncertainty propagation distribution from the Monte Carlo algorithm, assessment of the conformity compliance of the artefact of calibration in particular measurement points will be made against prescribed decision making rules embedded in the MonteCalc Uncertainty Toolkit, [15].

II. CALIBRATION OF HIGH FREQUENCY INSTRUMENTS IN LEM

LEM is an accredited calibration laboratory for electrical quantities instruments according to ISO 17025:2017 since 2015, [17]. The accredited method for calibration of high frequency instruments (oscilloscopes, counters and function generators), [1], is in compliance to the Calibration Guide EURAMET cg-7, [4]. The reference standard for calibration of oscilloscopes, is a Multifunctional Calibrator with a option for calibration of high frequency instruments i.e., oscilloscopes and counters with frequency range up to 630 MHz, depicted in Figure 1, with technical specifications in [18]. It is calibrated in the oscilloscope measurement range at the producer's accredited calibration laboratory with established measurement traceability to national (NPL) and international primary reference standards (BIPM), with an accompanying calibration certificate.

Figure 1. LEM reference standard - multifunctional calibrator with oscilloscope calibration option

The calibration guide EURAMET cg-7 prescribes an oscilloscope calibration procedure including two phases:
- vertical deflection calibration (voltage amplitude measurement along the vertical axis), and
- frequency bandwidth calibration (frequency measurement along the horizontal axis).

The two stages are independent from each other, but both are required to perform a full oscilloscope calibration procedure. According to the LEM calibration procedure, and in compliance with the calibration guide EURAMET cg-7, each set value, generated by the reference standard, is measured 12 times repeatedly. The validity of the

developed calibration procedure is confirmed by applying it to a real case scenario. The artefact of calibration (unit under test-UUT) is a 3 GHz 4-channel Oscilloscope, in Figure 2 and with technical specification in [19]. The UUT is high accuracy class test equipment for the telecommunications a with a high frequency bandwidth.

Figure 2. UUT - 3 GHz 4-channel oscilloscope calibrated with the LEM reference standard

To obtain the uncertainty budget for the two calibration stages, data from a component of type A and components of type B are fused, following the guidelines of GUM [9], by deploying the original LEM data fusion concept as in [16], and complying with the recommendations of the Euramet guide, [4]. The type A uncertainty, u_A, is derived from the statistical analysis of the experimental data, namely the mean value, X_{mean}, and the standard deviation of the measurement, s_A, as shown in:

$$s_A = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_{icor} - X_{mean})^2} \qquad (1)$$

where

$$X_{mean} = \frac{1}{n}\sum_{i}^{n} X_{icor} \qquad (2)$$

$$X_{icor} = X_i - X_{ref} \qquad (3)$$

X_i is the measured value in the particular measurement point and X_{ref} is the reference value from the calibrator. The following uncertainty components are fused, in the uncertainty budget of type B, u_B:

u_{res_instr} – from the calibrated instrument resolution,
u_{res_refst} – from the reference standard resolution,
u_{d_refst} – from the reference standard drift,
u_{c_refst} – from the reference standard calibration.
The combined uncertainty of type B equals:

$$u_B = \sqrt{u_{res_{instr}}^2 + u_{res\_refst}^2 + u_{d\_refst}^2 + u_{c\_refst}^2} \qquad (4)$$

The total combined uncertainty is:

$$u_c = \sqrt{u_A^2 + u_B^2} \qquad (5)$$

The expanded uncertainty deployed in the rule for conformity statement is:

$$U = 2 \cdot u_c \qquad (6)$$

III. ESTIMATION OF UNCERTAINTY IN CALIBRATION OF OSCILLOSCOPE BY USING MONTECALC UNCERTAINTY TOOLKIT

The originally developed software package in LEM for

Monte Carlo simulation entitled MonteCalc Uncertainty Toolkit, [15] features the following functionalities:
- automatic calculation of the measurement uncertainty budget based on the parameters entered in Set Up according to GUM methodology,
- automatic calculation of the compliance graphs depending on a limit value that is set with an appropriate control,
- automatic calculation of Monte Carlo simulation depending on the parameters entered in Set Up,
- selection of the number of random variables, and distribution partitions,
- automatic calculation of fit graphs depending on a threshold value that is set with appropriate control,
- saving the final distribution and fit graphs in .jpeg format, and the simulation results in .csv and
- automatic scaling of y and x axes of fit graphs in order to obtain greater visibility of results.

The MonteCalc Uncertainty Toolkit is universal and can be applied to any case of testing/calibration uncertainty estimation, with no restrictions on the type of instrument, the physical quantity measured, the range, the type probability distribution etc. It is developed in the graphical programming language LabVIEW[TM], [20], and the executive file is denoted as virtual instrument. The Monte Carlo simulation is performed using a virtual instrument palette that allows the generation of random numbers with an appropriate distribution. The generation of the distribution in this case is done depending on the Type B parameters that are entered. For each Type B

component, a distribution of random numbers is generated and summed together with the Type A component. From the resultant distribution, the uncertainty of the Monte Carlo simulation is calculated. The MonteCalc Uncertainty Toolkit has also an incorporated complex evaluation module for conformity assessment against different prescribed decision-making rules in line with the international Guideline ILAC G8:09/2019, [21]. The built-in decision-making rules in MonteCalc are the binary rule, the binary rule with guard band, the non-binary rule with guard band and with measurement uncertainty, and other.

IV.   RESULTS AND DISCUSSION

The results presented in this contribution are only selected specific calibration points from the overall calibration protocol of the UUT. For the vertical deflection calibration, the selected point is the 2 V/Div Channel 3 of the UUT.

Table 1 Comparison of the extended measurement uncertainty calculated according to the GUM and Monte Carlo at a calibration point along the vertical axis of 2 V/Div for the Channel 3 of the UUT

Expanded measurement uncertainty GUM	Expanded measurement uncertainty Monte Carlo simulation
0,02 V	0,05 V
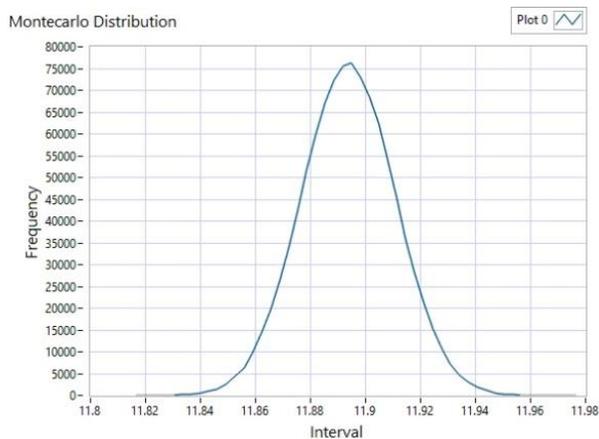
Fig. 3 Probability distribution obtained by MonteCalc Uncertainty Toolkit of the vertical deflection calibration at point of 2 V/Div Channel 3 of the UUT

In Figure 3 is the probability distribution obtained by MonteCalc Uncertainty Toolkit, while in Table 1 are the measurement uncertainties derived from the Toolkit, by applying the GUM approach and by conducting the Monte Carlo simulation, respectively. The derived calibration results with the accompanying uncertainty are subjected to conformity assessment by the built-in option of the

MonteCalc Uncertainty Toolkit, against different decision making rules.

In Figures 4-7, the compliance graphs obtained by the Monte Carlo simulation method depending on the limit value set with an appropriate control for the particular decision making rule, i.e. guard band *w*, at the calibration point of 2 V/Div Channel 3 of the UUT, are displayed.

Fig. 4 Measured result at a calibration point along the vertical axis 2 V/Div for UUT (Channel 3) for w=0,83U

Fig. 5 Measured result at a calibration point along the vertical axis 2 V/Div for UUT (Channel 3) for w=U

Fig. 6 Measured result at a calibration point along the vertical axis 2 V/Div for UUT (Channel 3) for w=1,5U
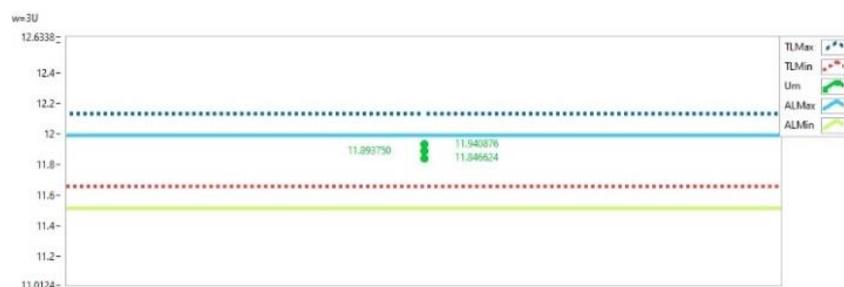

Fig. 7 Measured result at a calibration point along the vertical axis 2 V/Div for UUT (Channel 3) for w=3U

In Table 2, the acceptance criteria are expressed, according to each decision making rule at the same calibration point. The following symbol legend is applied:
- ✓ - accepted
- ☑ - conditionally accepted
- ☒ - conditionally rejected
- ✗ - rejected.

Table 2 Declarations of conformity at a calibration point on the vertical axis 2 V/Div for UUT Channel 3 based on the outcomes for different decision-making rules

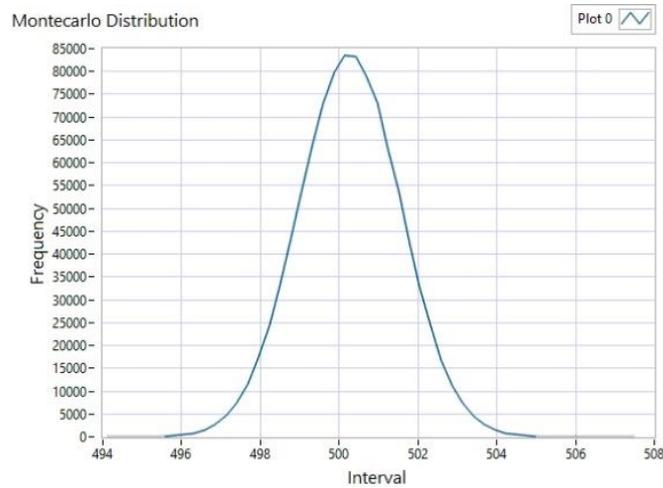| Decision rule w	$0,83U$	U	$1,5U$	$3U$
Binary	✓	✓	✓	✓
Binary with guard band	✓	✓	✓	✓
Non-binary with guard band	✓	✓	✓	✓
Non-binary with guard band & measurement uncertainty	✓	✓	✓	✓

Fig. 8 Probability distribution obtained by MonteCalc Uncertainty Toolkit of the frequency bandwodth of 500 MHz Channel 2 of the UUT

For the frequency bandwidth calibration the selected point is the 500 MHz Channel 2 of the UUT.

In Figure 8 is the probability distribution obtained by MonteCalc Uncertainty Toolkit, while in Table 3 are given the measurement uncertainties derived from the Toolkit, by applying the GUM approach and by the Monte Carlo simulation, respectively.

Table 3 Comparison of the extended measurement uncertainty calculated according to the GUM and Monte Carlo at a calibration point at the frequency bandwidth of 500 MHz Channel 2 of the UUT

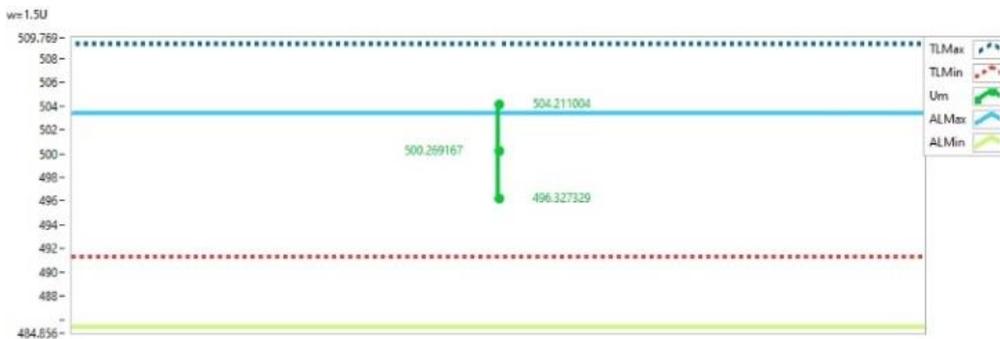| Expanded measurement uncertainty GUM	Expanded measurement uncertainty Monte Carlo simulation
2,298 MHz	3,94 MHz

Figure 9 Result at a calibration point of the frequency bandwidth of 500 MHz of the UUT (Channel 2) for w=1,5U
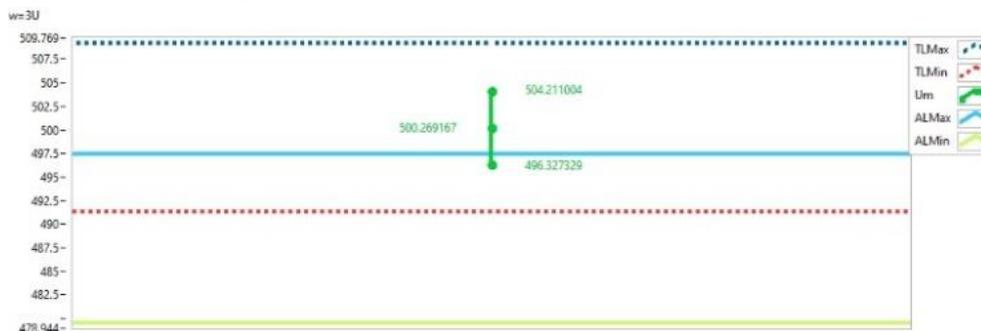

Fig. 10 Result at a calibration point of the frequency bandwidth of 500 MHz of the UUT (Channel 2) for w=3U

In Figures 9 and 10, the compliance graphs obtained by the Monte Carlo method depending on the limit value set with an appropriate control for a particular decision making rule, i.e. guard band w, at the calibration point of 500 MHz of the UUT (Channel 2), are displayed. In Table 4, the acceptance criteria are expressed, according to each decision making rule at the same calibration point. The same symbol legend as in Table 2 is applied.

Table 4 Declarations of conformity at a calibration point of the frequency bandwidth of 500 MHz of the UUT (Channel 2) according to the outcomes for different decision-making rules

Decision rule w	$0{,}83U$	U	$1{,}5U$	$3U$
Binary	✓	✓	✓	✓
Binary with guard band	✓	✓	✓	✗
Non-binary with guard band	✓	✓	☑	☒
Non-binary with guard band & measurement uncertainty	✓	✓	✓	✓

The two methods yielded noticeably different uncertainty values, which is not surprising as the GUM assumes a normal distribution, an assumption that is not fully true in case of oscilloscopes calibration. In contrast, the Monte Carlo simulation accounts for the actual distribution by randomly generating variable values. So, the Monte Carlo approach is more appropriate, as it produces a broader uncertainty range, thus supporting more conservative and reliable conformity assessments.

V. CONCLUSIONS

The LEM-developed software, MonteCalc Uncertainty Toolkit, which implements both the standard GUM and the Monte Carlo method for evaluating measurement uncertainty, was successfully used on a real-world laboratory calibration of a high-frequency instrument – an oscilloscope. The uncertainty results obtained served as inputs for the decision-making process, which is integrated as a module within the MonteCalc Uncertainty Toolkit. The satisfactory results confirm the effectiveness of the developed software approaches.

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