

Reactive power/energy instruments in harmonically distorted conditions – analysis of different measuring algorithms’ response in relation to fundamental reactive power

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Abstract – The measurement of reactive power/energy in non-sinusoidal conditions presents several challenges. The primary issue is the ambiguous definition of reactive power in the presence of high order harmonics. International standard IEEE 1459 addresses this by emphasizing fundamental power components for system monitoring, even in polluted conditions. Another challenge is the existence of different measuring algorithms, implemented in modern devices, which yield consistent response for sinusoidal waveforms, but differ under harmonically distorted voltages and currents. This study experimentally verifies the readings of various meters for reactive power/energy against mathematically modeled outputs. The reference quantity for analysis is fundamental reactive power, as defined in Std. IEEE 1459. The experiments include both single-harmonic and randomly distorted signals, with error analysis based on the share of fundamental reactive power. To ensure high metrological accuracy, the study employs reference standards of the highest accuracy class, traceable to BIPM intrinsic standards.

Keywords: high order harmonics, fundamental reactive power/energy, measurement error.

I. INTRODUCTION

The requirements for active power/energy measurement, in harmonically distorted conditions, are covered in international standards [1-3], recommendation [4] and many scientific works [5-8]. In the existing standards and recommendation, test signals for examination of the concrete measuring equipment, under non-sinusoidal conditions, are proposed and error limitations, regarding different accuracy class instruments, are presented [3-4]. This is due to the fact that the active power is an electrical quantity that is unambiguously defined, even in the presence of high order harmonics in the input signals.

The situation is quite the opposite in case of reactive power and energy measurements, for two main reasons.

The first one is the fact that the reactive power is a quantity that is ambiguously defined in case of non-sinusoidal waveforms, i.e. nowadays multiple power definitions exist, each one possessing certain advantages and flaws [9-12]. For example, the power theory proposed by *Budeanu* [9] is regarded as a basic frequency-based approach for determination of the aforementioned quantity in case of distorted signals. The definition according to *Fryze* [10] is referred to as a basic time-domain approach, for representing the reactive power in case of harmonics. Additionally, commercially available instruments in present days possess different measuring algorithms that provide the same response in case of pure sinusoidal voltages and currents, but the output may be completely different in case of distorted signals. In the international standards EN 62053 [13-14], the accuracy demands for different class static reactive electricity meters are presented, but they are limited to sine-wave conditions.

An effort for overcoming the specific issues was made with the publication of IEEE 1459 standard [15], in which the fundamental power components are stated to be of particular interest when it comes down to system parameters monitoring. However, the measurement of fundamental reactive power in case of harmonically distorted signals, may require alteration of the existing measuring solutions [16-17]. In terms of legal metrology issues, the measurement of fundamental reactive power/energy provides inequality in terms of billing penalization of harmonic distortion producers and consequently compensation for the distorted energy consumers [17].

Taking into account all previously mentioned complications, an analysis of the recordings made by different instruments for reactive power/energy, in case of harmonically distorted input signals, will be performed. The instruments’ output is going to be compared with a mathematically modelled deviation between the measured quantity and the reference fundamental reactive power, as proposed in [15]. In order for the measurements to be conducted at the highest metrological level, reference

standards (RSs) of the highest accuracy class, will be used, which are traceable to the intrinsic standards of BIPM [18].

II. REACTIVE POWER/ENERGY MEASURING ALGORITHMS

In the discussion that follows, two most widely used measuring solutions, for reactive power/energy monitoring, will be presented. The first one is based on phase shifting of the voltage (or current) signals by 90° , before their multiplication with the instantaneous current (or voltage) signals. The phase shifting is conducted before the digitalization of the signals, and therefore the concrete measuring algorithm is addressed to as a concept based on analogue (phase) displacement of the input signals. The phase shifting of the signals is accomplished by means of integration circuit. [19-23]. In case of harmonically distorted input signals, the measured quantity, Q_M , may be presented with the following equation [19-23]:

$$Q_M = \frac{1}{T} \int_0^T \frac{2\pi}{T} \left[- \int u(t) dt \right] i(t) dt = \sum_{h=1}^n \frac{U_h I_h \sin \varphi_h}{h}, \quad (1)$$

where $u(t)$ and $i(t)$ are the measured voltage and current, expressed in time domain, U_h and I_h are the RMS values of the high order harmonics of voltage and current signals of order h , φ_h is the phase shift between them, and T is the period of the signals' components with fundamental frequency.

The second measuring algorithm is based on time displacement of the voltage (or current) signals' samples for one quarter of a period, $T/4$, regarding the fundamental signals' frequency, before their multiplication with the corresponding current (or voltage) signals' samples [19-23]. This process is conducted after the analogue to digital conversion of the input signals and therefore it is referred to as digital (time) displacement approach. The measured quantity, in case of harmonically distorted input signals, equals [19-23]:

$$Q_M = \frac{1}{N} \sum_{j=m}^{m+(N-1)} u_j \cdot i_{j-(T/4)} = \sum_{h=1}^n \pm U_h I_h \sin \varphi_h (\cos \varphi_h), \quad (2)$$

where u_j and i_j are the voltage and current input signals' digital samples taken for averaging in a so called "averaging window", consisting of N samples, and all other parameters, U_h , I_h , φ_h , possess the same meaning as described above. According to [19-23] the even harmonics' power fractions, are regarded as active power components, while the odd harmonics' power fractions are recorded as reactive power components, with different sign that is dependent on their order, h .

The reference quantity against which the instruments' output will be analyzed is the fundamental reactive power:

$$Q_1 = U_1 I_1 \sin \varphi_1, \quad (3)$$

where U_1 and I_1 are the voltage and current components at 50 Hz and φ_1 is the phase shift between them.

Equations (1)-(3) are presented for single phase conditions, if a three phase system is regarded, both the measured and the reference power are calculated as a sum of the power readings in all three phases [24]. In a special case scenario, where three phase symmetrical conditions are applied, both the measured and the reference three phase reactive powers are calculated by multiplying (1)-(3) with a factor of 3. It is important to be emphasized however that three phase symmetrical conditions in case of harmonically distorted signals imply the same amplitude and phase shifts for all three phase voltages and currents, regarding both fundamental components and high order harmonics.

Regarding the presentation of high order harmonics in the input voltages and currents, the concept of relative share of each harmonic is used [3-4, 24]:

$$x_{h,\%} = \frac{X_h}{X_1} \cdot 100, \quad (4)$$

where X_h is the RMS value of the h^{th} order harmonic in voltage or current signals, while X_1 is the RMS of the corresponding component at 50 Hz. The phase shift of any harmonic of order h is presented in relation to the initial phase shift of the voltage/current fundamental, α_{x1} , at positive zero crossing as [3-4]:

$$\theta_{xh} = \angle(\alpha_{xh}, \alpha_{x1}), \quad (5)$$

where α_{xh} is the initial phase shift of the h^{th} order harmonic.

III. MEASUREMENT EQUIPMENT AND TEST PROCEDURES

The experimental part of the work is conducted in Laboratory for Electrical Measurements (LEM), within the Faculty of Electrical Engineering and Information Technologies (FEEIT), at Ss. Cyril and Methodius University in Skopje (UKIM). It is an accredited laboratory for calibration of instruments for electrical quantities, according to the international standard MKC EN ISO/IEC 17025:2018 [25], and it maintains international traceability to the intrinsic RSs of BIPM [18].

For the needs of the following experiment, the secondary RS of LEM, in domain of AC electrical power and energy instruments calibration, CALMET C300 [26], is used as source of harmonically distorted signals, as well as a reference unit for fundamental reactive power/energy.

A reactive energy electricity meter of accuracy class 1, Landys+Gyr ZMD405CT44.2407 [27], is used as a Unit Under Test (UUT). The measuring algorithm of this UUT is based on analogue (phase) displacement of three phase voltages by 90° . LEM's primary RS in domain of AC power and energy instruments calibrations, ZERA COM3003 [28], is also used, taking into account that its

measuring algorithm is based on digital (time) displacement of the current signals. By applying tests with harmonically distorted signals, their outputs based on (1) and (2) respectively will be verified and error analysis, in relation to fundamental reactive power, will be performed.

For the experimental part of the work, two sets of test signals are generated from the reference source [26]. The first one is denoted as 5H test, taking into account that it is based on the test signals proposed in [3], used for verification of active electricity meters in distorted conditions. The waveforms consist of 5th order harmonics, beside fundamental voltages and currents, with a relative share of 10 % and 40 %, respectively. The 5th order voltage harmonic is in phase with the voltage fundamental at positive zero crossing, while the 5th order current harmonic is phase shifted in relation to the fundamental current by 60°. The second test signals set is referred to as RANDOM test, due to the random distortion of the waveforms, with several pre-applied constraints. Namely, test signals are comprised of odd harmonics only, up to 11th order, and the total harmonic distortion is limited at 10 %, for the voltage signals, and 40 %, for the current signals. Both tests are performed in three phase symmetrical conditions and single RMS value voltages and currents are used. For the electricity meter [27] they are equal to the meter's nominal parameters, 58 V and 5 A, respectively, while for the primary RS [28], they equal 220 V and 5 A, respectively. Measurements in twelve measuring points that correspond to different fundamental phase shift, φ_1 , are conducted, ranging between -90° and -15°, with a step of 15°, and between 15° and 90°, once again with a step of 15°.

IV. TEST RESULTS AND DISCUSSION

According to (1) and (3), when the electricity meter [27] is subjected to harmonically distorted test signals, the relative deviation between its readings and the reference fundamental reactive power/energy may be evaluated as:

$$\begin{aligned} \varepsilon_{1,\%} &= \frac{Q_{M,3f} - Q_{1,3f}}{Q_{1,3f}} \cdot 100 = \frac{\sum_{h>1}^n 3 \cdot \frac{U_h I_h \sin \varphi_h}{h}}{3U_1 I_1 \sin \varphi_1} = \quad (6) \\ &= \sum_{h>1}^n \frac{1}{h} \cdot \frac{u_{h,\%}}{100} \cdot \frac{i_{h,\%}}{100} \cdot \frac{\sin \varphi_h}{\sin \varphi_1} \cdot 100, \end{aligned}$$

which indicates that the error is directly proportional to the share of single harmonics in the test signals, (4), $u_{h,\%}$ and $i_{h,\%}$, and it is inversely proportional to the harmonics' order, h . The error is dependent on the phase shift between harmonic voltages and currents, which is calculated as [7]:

$$\varphi_h = h\varphi_1 - (\theta_h - \theta_{uh}), \quad (7)$$

where θ_{uh} and θ_h are high order harmonics initial phase shifts in relation to fundamental components, as depicted in (5) and it is inversely proportional to the fundamental

reactive power share in the system, i.e. $\sin \varphi_1$.

The results from the examination of the electricity meter [27] for both sets of test signals are depicted in Fig.1 and Fig.2. In both figures, two error curves are given. The first one, denoted as $\varepsilon_{1,\%(\text{meas})}$, represents the errors obtained during the experimental examination of the UUT. The second error curve, $\varepsilon_{1,\%(\text{math})}$, represents the mathematically modelled deviations, by using (6). Both Fig.1 and Fig.2 indicate that the measured deviations follow the envelope of the mathematically modelled errors, in such a manner the modulus operandi of the UUT, according to (1), is verified. An error mismatch in some measurement points exist, and it is assumed to be a result of the existing actual asymmetry between the three phase voltages and currents and the intrinsic errors [4] of the meter. As far as the error intensities are regarded, it may be concluded that they are relatively small, in case of high fundamental reactive power share in the system. When φ_1 is between $\pm 60^\circ$ and $\pm 90^\circ$ the meter's actual performance is within the accuracy limits (class 1). For lower φ_1 , errors up to several percent are detected, a conclusion that is valid for both tests.

When LEM's primary RS [28] is subjected to the introduced test signals, the errors between the measured and the reference fundamental reactive power may be mathematically evaluated as follows:

$$\begin{aligned} \varepsilon_{1,\%} &= \frac{Q_{M,3f} - Q_{1,3f}}{Q_{1,3f}} \cdot 100 = \frac{\sum_{h>1}^n \pm 3U_h I_h \sin \varphi_h (\cos \varphi_h)}{3U_1 I_1 \sin \varphi_1} = \quad (8) \\ &= \sum_{h>1}^n \pm \frac{u_{h,\%}}{100} \cdot \frac{i_{h,\%}}{100} \cdot \frac{\sin \varphi_h (\cos \varphi_h)}{\sin \varphi_1} \cdot 100, \end{aligned}$$

taking into account its measuring algorithm, based on (2).

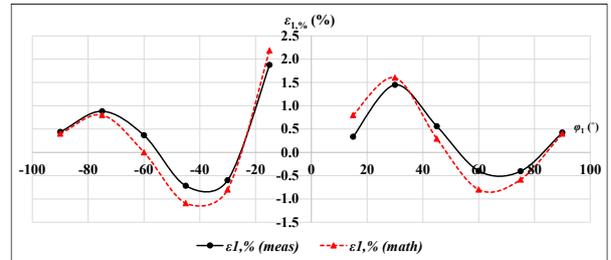


Fig. 1. Electricity meter's relative errors in relation to $Q_{1,3f}$ for the 5H test signals

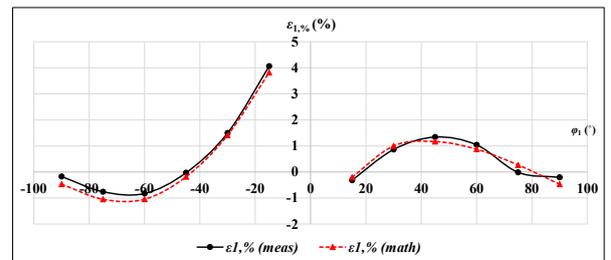


Fig. 2. Electricity meter's relative errors in relation to $Q_{1,3f}$ for the RANDOM test signals

If equations (6) and (8) are compared, it may be assumed that the errors in case of 5th order harmonics test, recorded when LEM's primary RS [28] is examined, will be 5 times higher than the recorded and mathematically evaluated deviations in case of using the electricity meter [27] as UUT. The concrete assumption is validated by using the results illustrated in Fig.3, where both the recorded errors curve, $\varepsilon_{1,\%}(\text{meas})$, and mathematically modelled deviations, $\varepsilon_{1,\%}(\text{math})$, provide justification of the point. The results presented in Fig.3 indicate that the errors in relation to fundamental reactive power, when an instrument based on digital (time) displacement of the test signals is used, are high enough that the corresponding meter may not be used by any means for measurement as demanded in the Std. IEEE 1459 [15]. For the concrete 5H test, errors up to $\pm 4\%$ are detected for high share of fundamental reactive power in the system. For fundamental phase shifts lower than $\pm 45^\circ$, the recorded deviations are up to more than 10 %.

Regarding the RANDOM test results, illustrated in Fig.4, it may be noted that lower overall errors are recorded, in comparison to the data presented in Fig.3. This is due to the fact that the UUT, based on (2), detects some odd harmonic power components with a positive sign, i.e. the way they actually flow in the system, while the others are detected with a negative sign, i.e. opposite the way they actually flow in the system [19-23]. Having both power fractions with a positive and negative sign, results in their mutual partial annulment, even though there is still harmonic power left which contributes to significant meter reading deviation in relation to the fundamental reactive power.

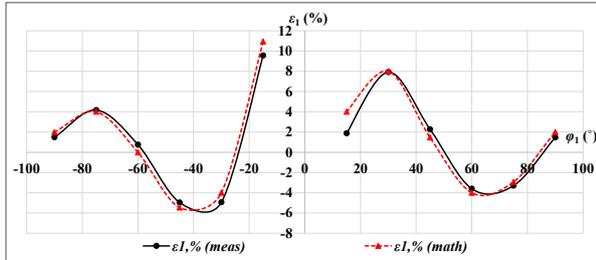


Fig. 3. LEM's primary RS relative errors in relation to $Q_{1,3f}$ for the 5H test signals

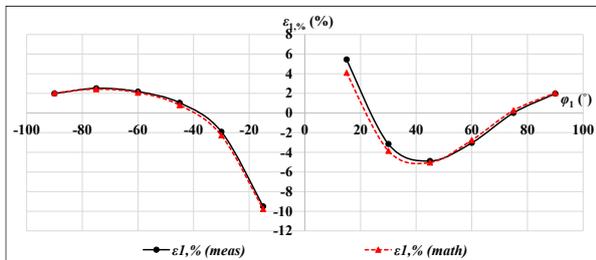


Fig. 4. LEM's primary RS relative errors in relation to $Q_{1,3f}$ for the RANDOM test signals

V. CONCLUSION

The results from the experimental examination of both instruments provide verification of the introduced measuring algorithms' output in case of harmonically distorted signals. The deviation between the modelled and the measured errors are smaller when the laboratory's primary RS's output is subject to analysis, taking into account its obviously better performance in comparison to the electricity meter, i.e. smaller intrinsic errors.

On the other hand, the instrument based on analogue (phase) displacement of the input signals, i.e. the electricity meter in the concrete analysis, provides better performance, in a scenario where it is supposed to record fundamental reactive power/energy. This is due to the fact that according to the measuring principle it records only fraction of the harmonic components' reactive power, as it circulates in the system, with no opposite sign regarding the power flow. For low distortion of the signals and high reactive power share in the system, the measured errors are found to be acceptable, as they lie within the limitations according to the accuracy class of the meter.

Finally, the different errors fluctuations and magnitudes, recorded for instruments based on alternative measuring algorithms, indicate that no direct comparison of their indications, within a calibration procedure in non-sinusoidal conditions, may be performed. An indirect way for establishing an unbroken traceability chain is possible however, by recording the relative share and phase shift of each harmonic with the primary RS and later providing the reference conditions for examination of the lower accuracy class meter in an analytical manner.

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